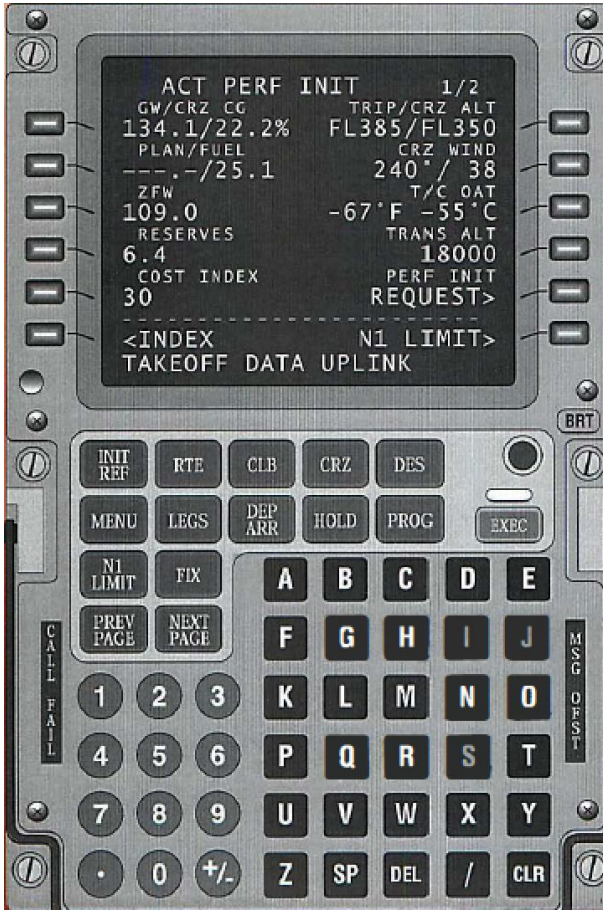
EE6900 Flight Management Systems

“Flight Management System Introduction”

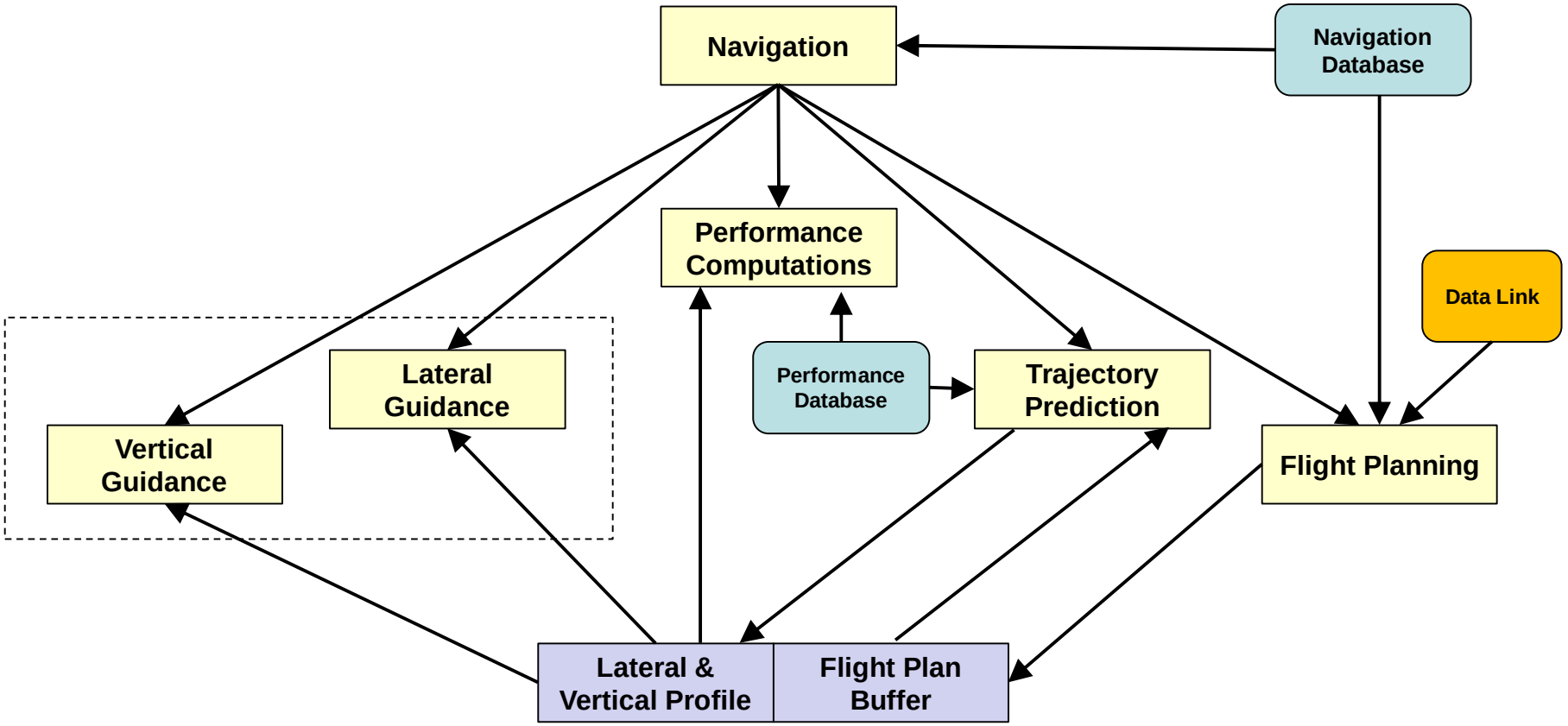
Dr. Maarten Uijt de Haag
Ohio University

Flight Management System (FMS)

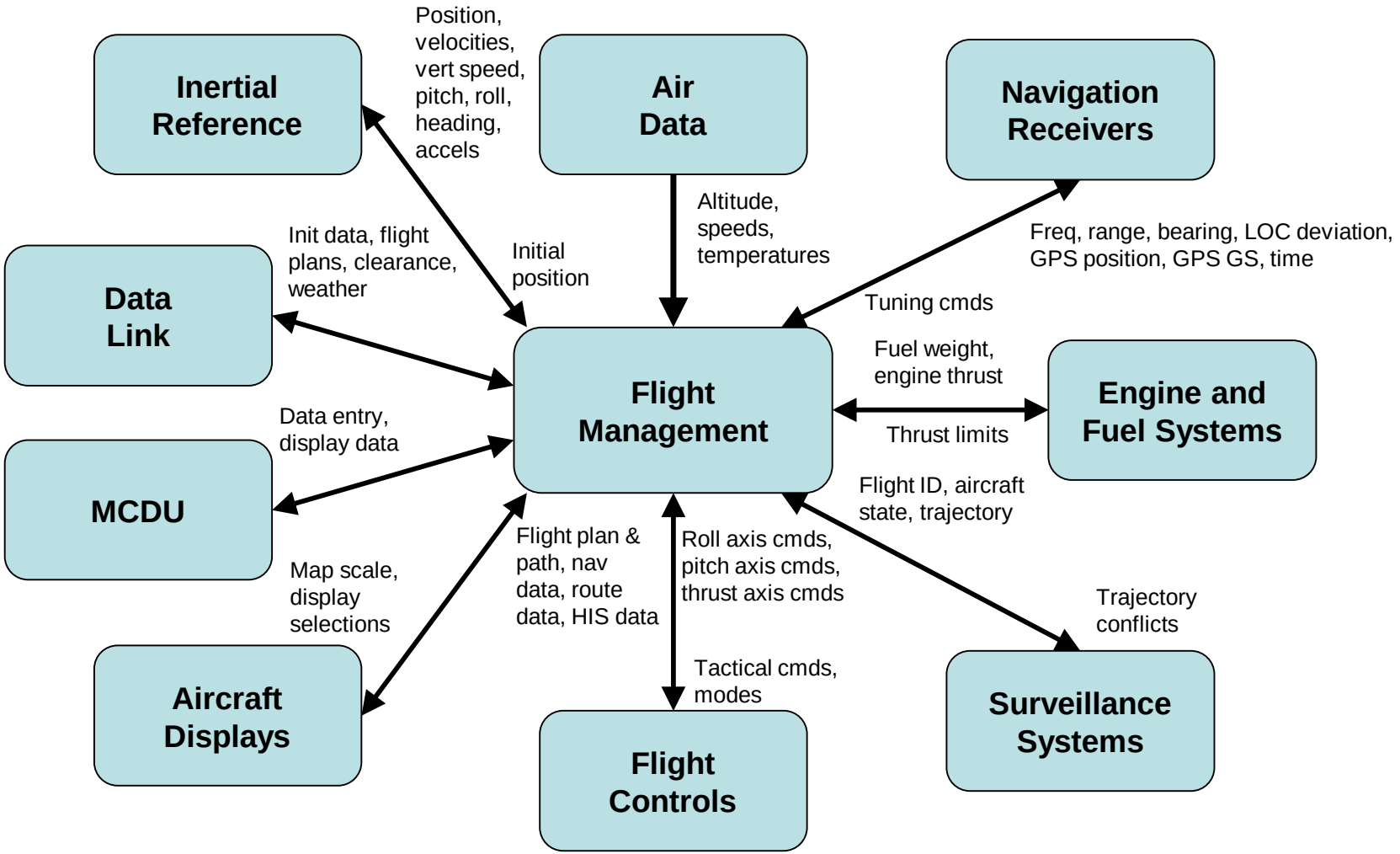
- Basic FMS functions:
  - **Navigation**
    - responsible for determining the best estimate of the current nav state of the aircraft.
  - **Flight planning**
    - allows the crew to establish a specific routing for the aircraft
  - **Trajectory prediction**
    - responsible for computing the predicted aircraft profile along the entire specified routing
  - **Performance computations**
    - provides the crew with aircraft unique performance information such as takeoff speeds, altitude capability, and profile optimization advisories
  - **Guidance**
    - responsible for producing commands to guide the aircraft along both the lateral and vertical computed profiles



FMS- Functional Block Diagram



Flight Management – Typical



Navigation - Navaids

Location	Avionics	Provides
Onboard	Air data (ADC)	pressure altitude, density altitude, OAT, TAS, EAS, AOA, etc.
	Inertial (INS)	attitude, true heading, magnetic heading, position, velocity, groundspeed, track etc.
Space-based	Global Positioning System (GPS)	position, velocity, groundspeed, track etc.
Terrestrial	Non-Directional Beacon (NDB)	bearing angle w.r.t. magnetic heading
	Distance Measuring Equipment (DME)	slant range
	Tactical Air Navigation System (TACAN)	slant range
	VHF Omnidirectional Range (VOR)	bearing angle w.r.t. magnetic North.
Landings	Instrument Landing System (ILS)	localizer and glideslope deviations
	Ground-Based Augmentation System (GBAS)	localizer and glideslope deviations (ILS) corrections, integrity indicators
	Microwave Landing System (MLS)	azimuth, elevation

Guest lecture next week on DME/DME!

Navigation – Navaid Symbols

Location	Avionics	Provides
Terrestrial	Non-Directional Beacon (NDB)	bearing angle w.r.t. magnetic heading
	Distance Measuring Equipment (DME)	slant range
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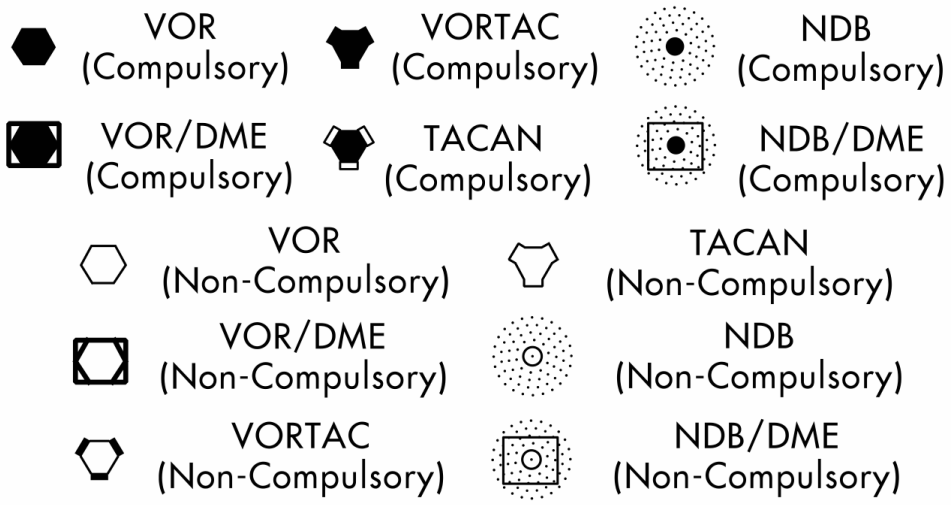
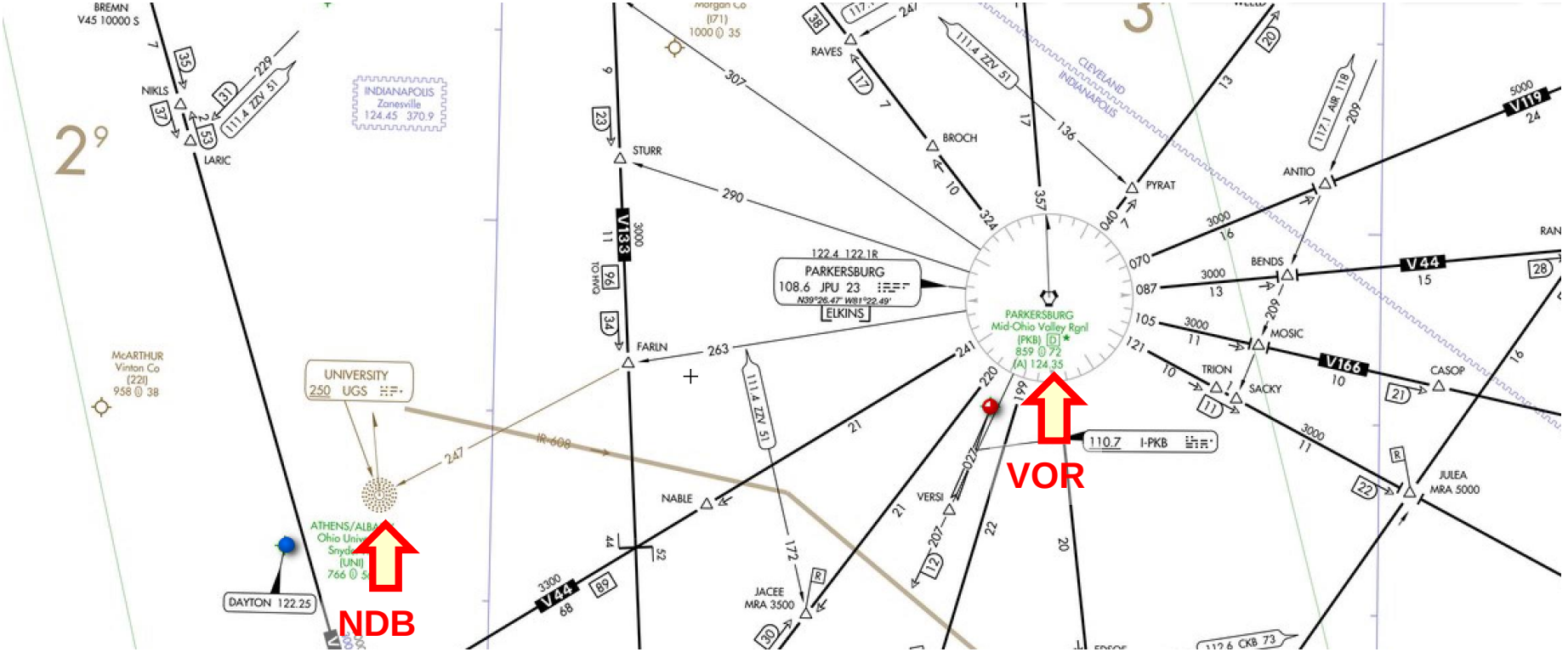
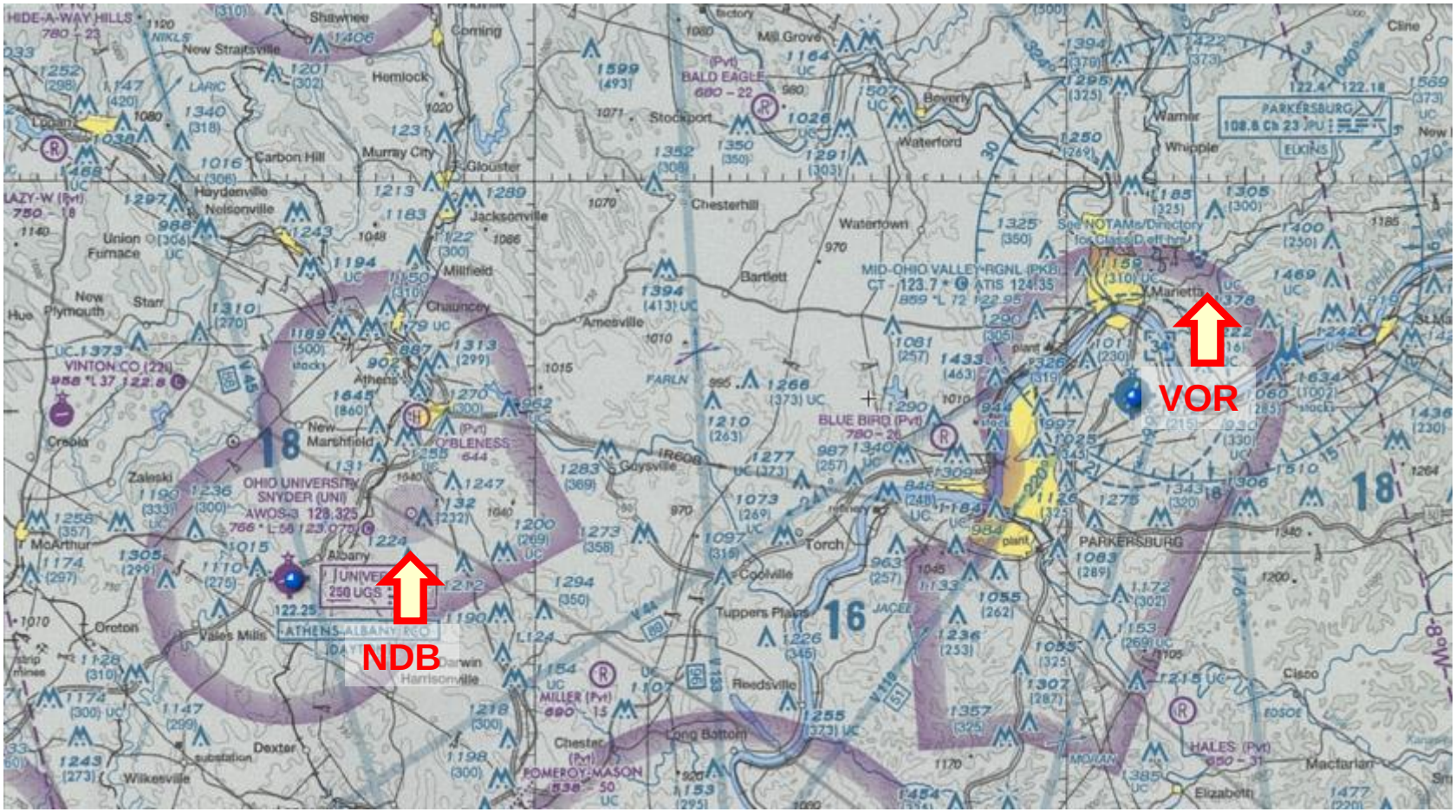


Low Altitude Enroute Chart

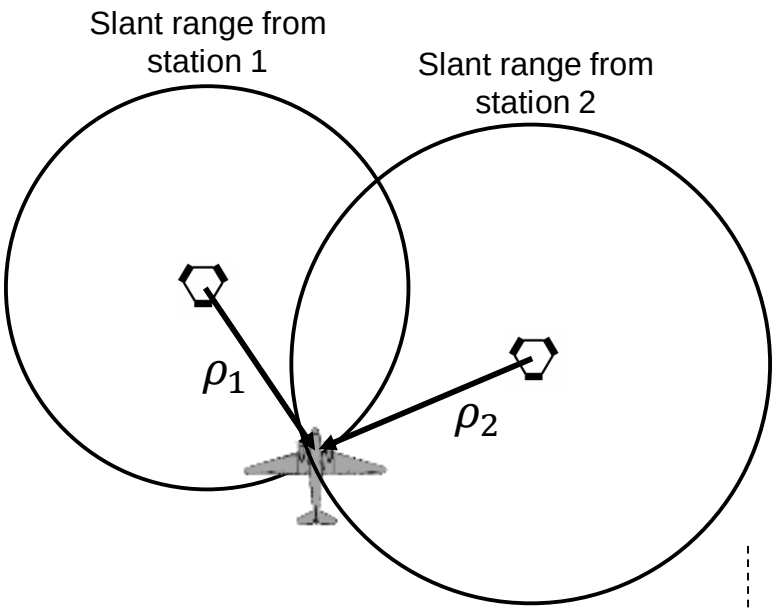


From: http://skyvector.com

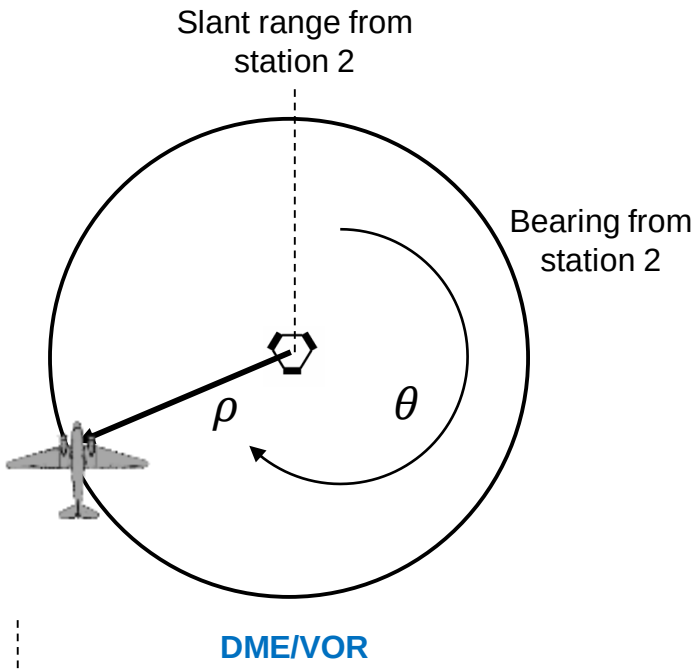
Navigation Charts - Sectional



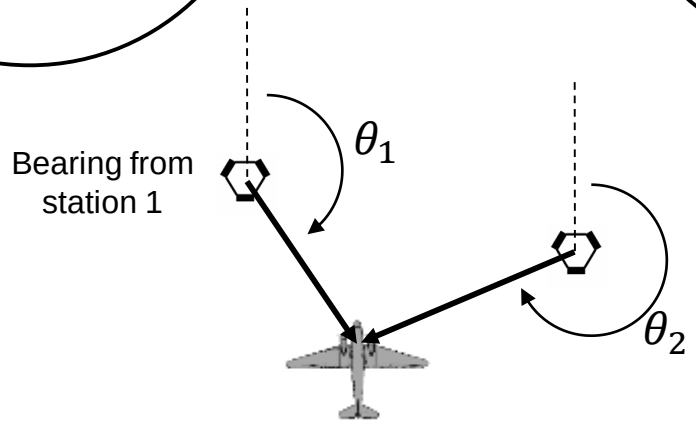
Navigation



DME/DME



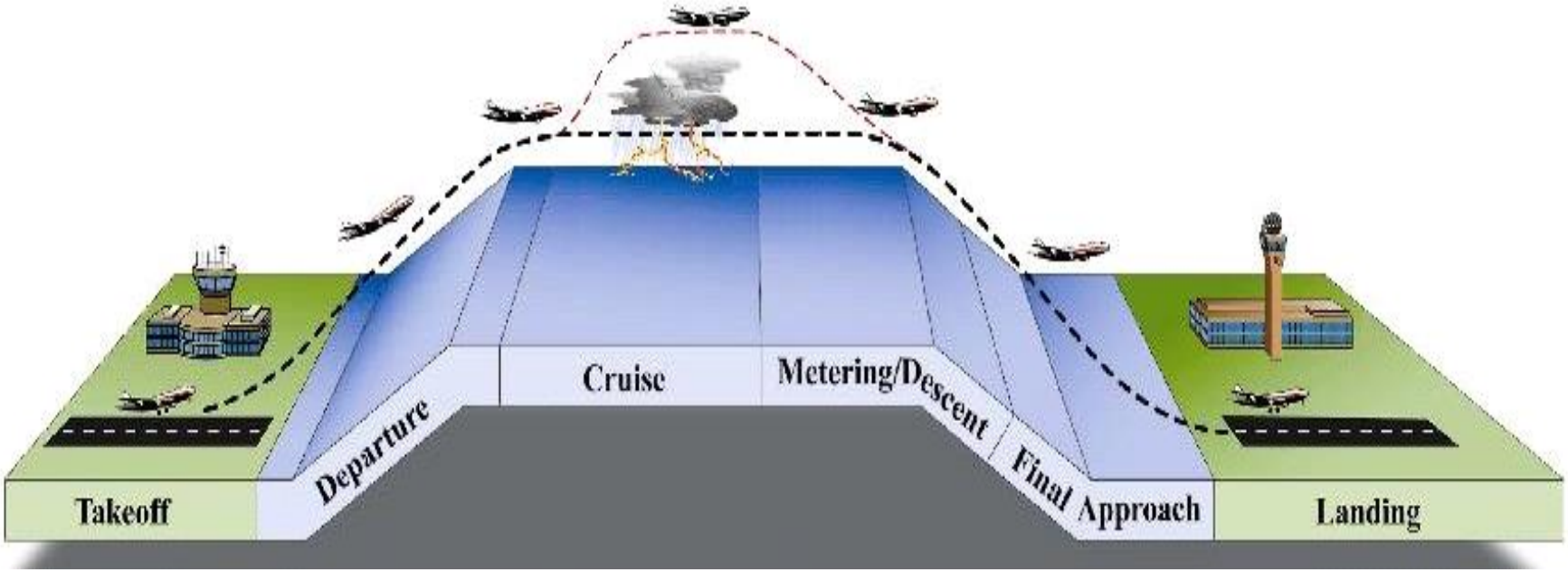
DME/VOR



VOR/VOR

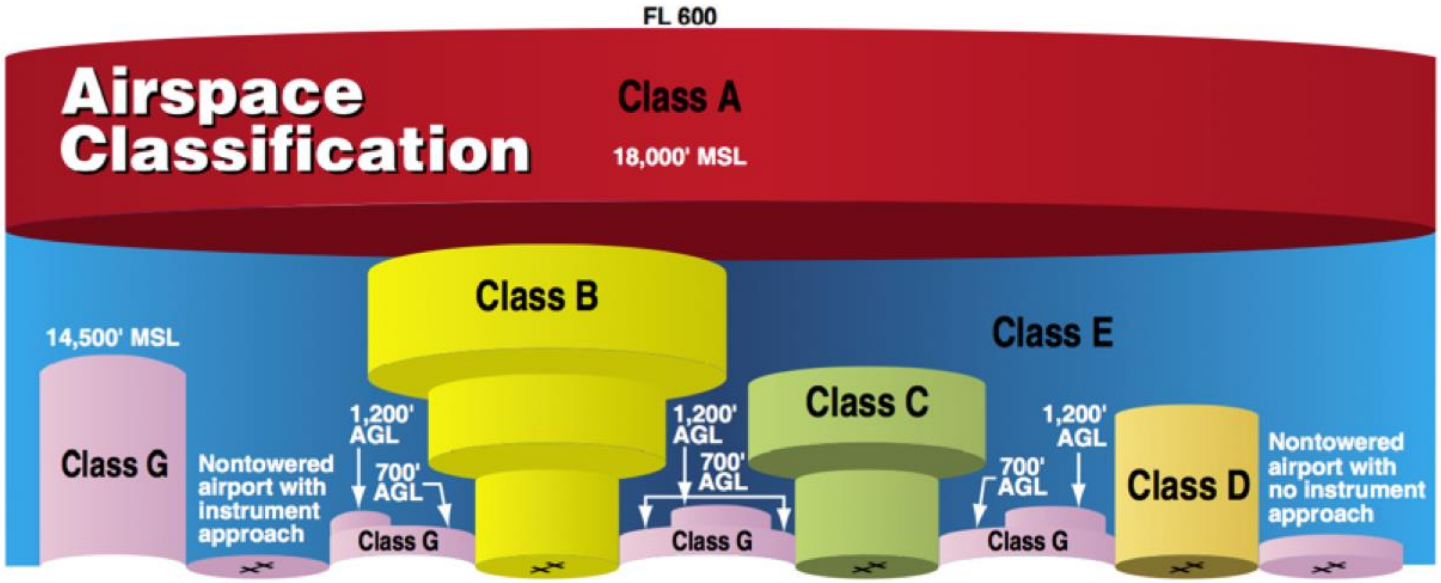
Based on symbol DME should really read TACAN.

Navigation – Phases of Flight



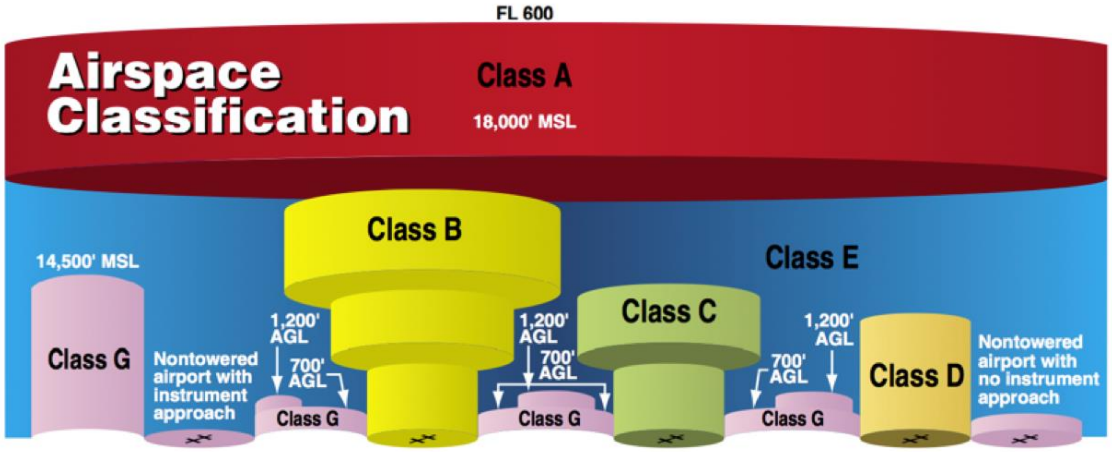
Terminal Area

Navigation - Airspace



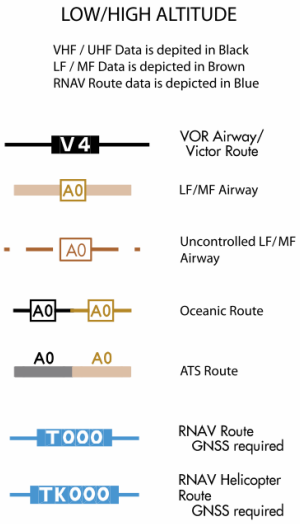
Airspace	Class A	Class B	Class C	Class D	Class E	Class G
Entry requirements	ATC clearance	ATC clearance	ATC clearance for IFR; all require radio contact	ATC clearance for IFR; all require radio contact	ATC clearance for IFR; all require radio contact	None
Minimum pilot qualifications	Instrument rating	Private of student certification.	Student certificate.	Student certificate	Student certificate	Student certificate
Two-way radio communications	Yes	Yes	Yes	Yes	Yes, under IFR flight plan	Not required
VFR Visibility Minimum	N/A	3 statute miles	3 statute miles	3 statute miles	< 10,000': 3 miles > 10,000': 5 miles	< 10,000': 1 (day)/3(night) miles > 10,000': 5 miles

Navigation - Routes



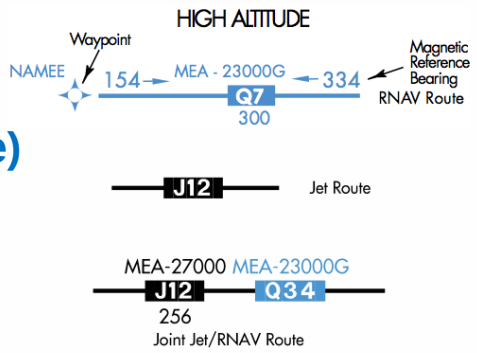
LOW ALTITUDE AIRWAYS

(in Class E airspace)

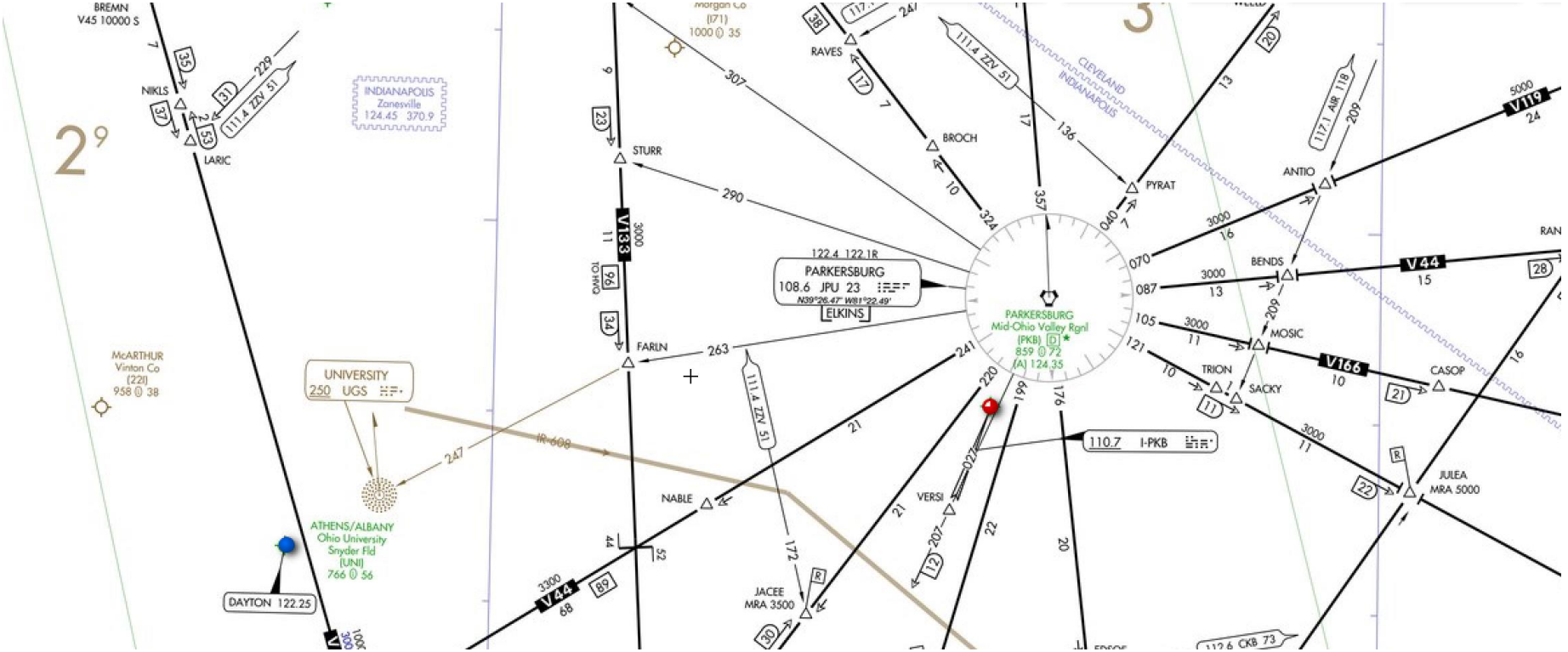


HIGH ALTITUDE ROUTES

(in Class A airspace)



Low Altitude Enroute Chart



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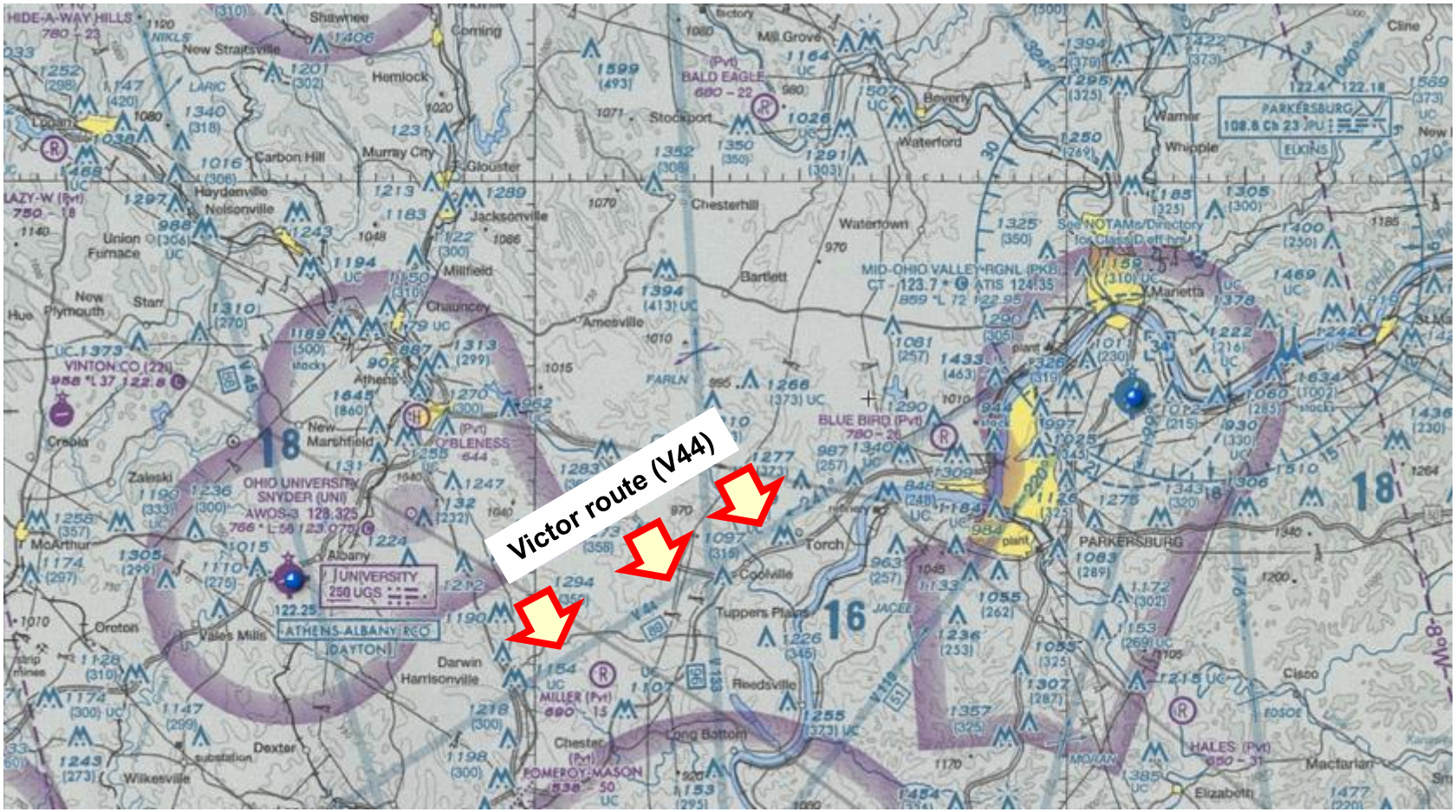
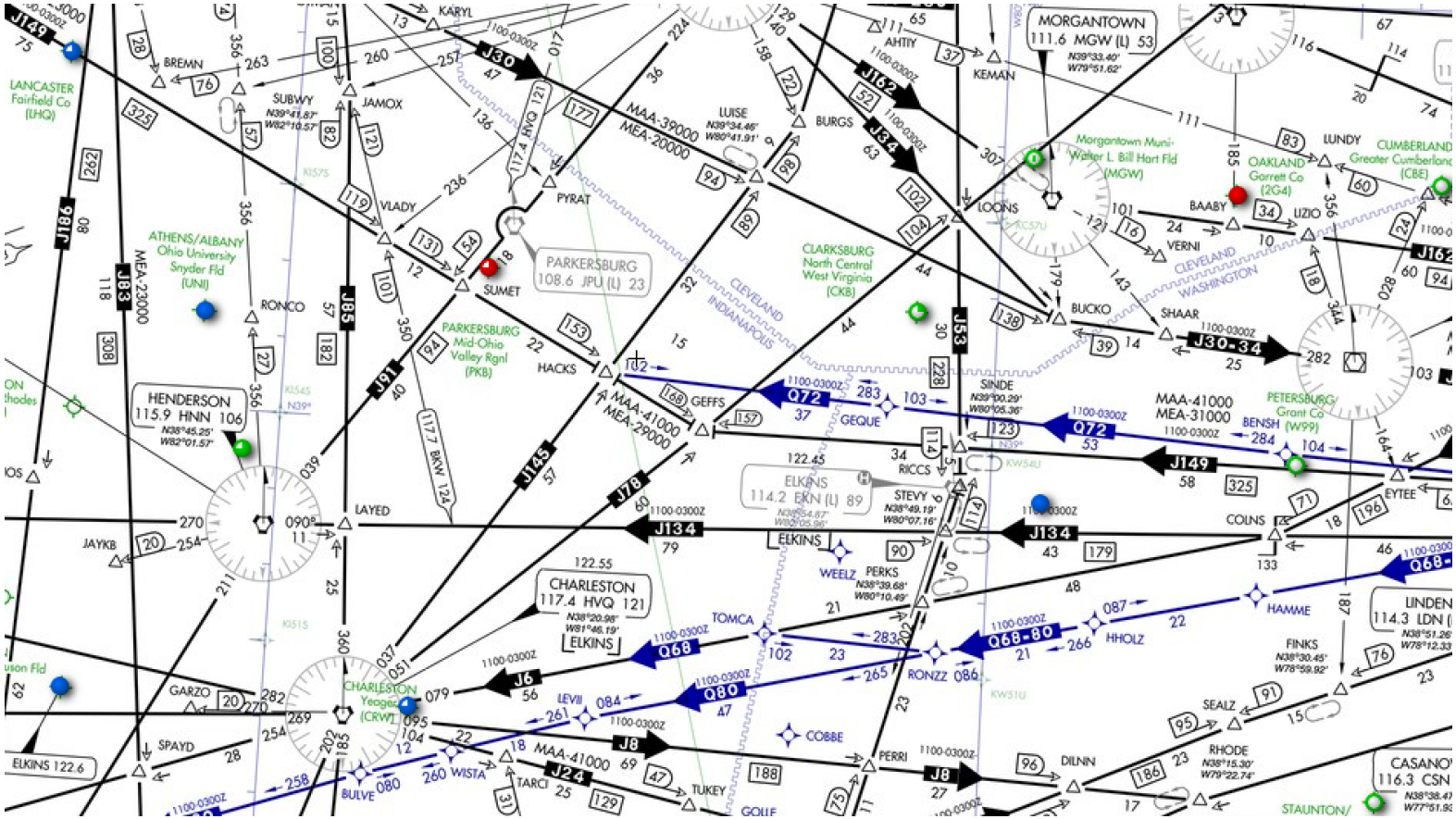
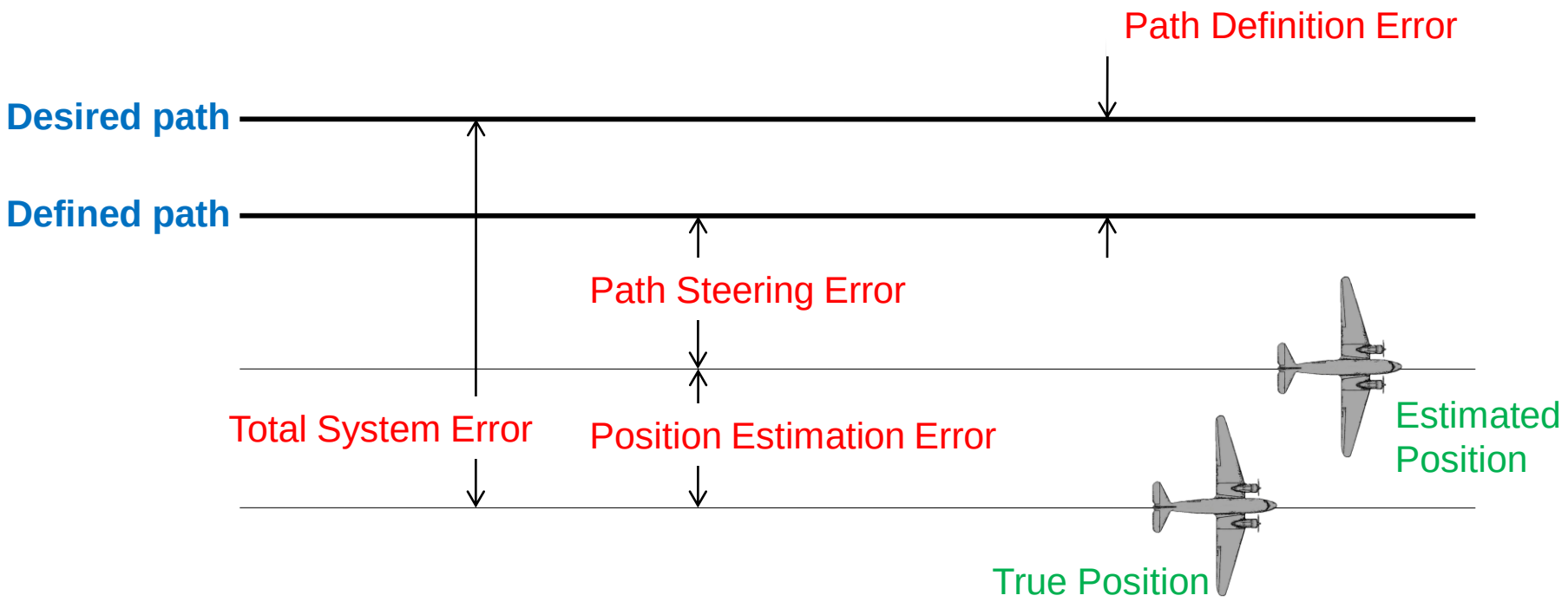


High Altitude Enroute Chart



From: http://skyvector.com

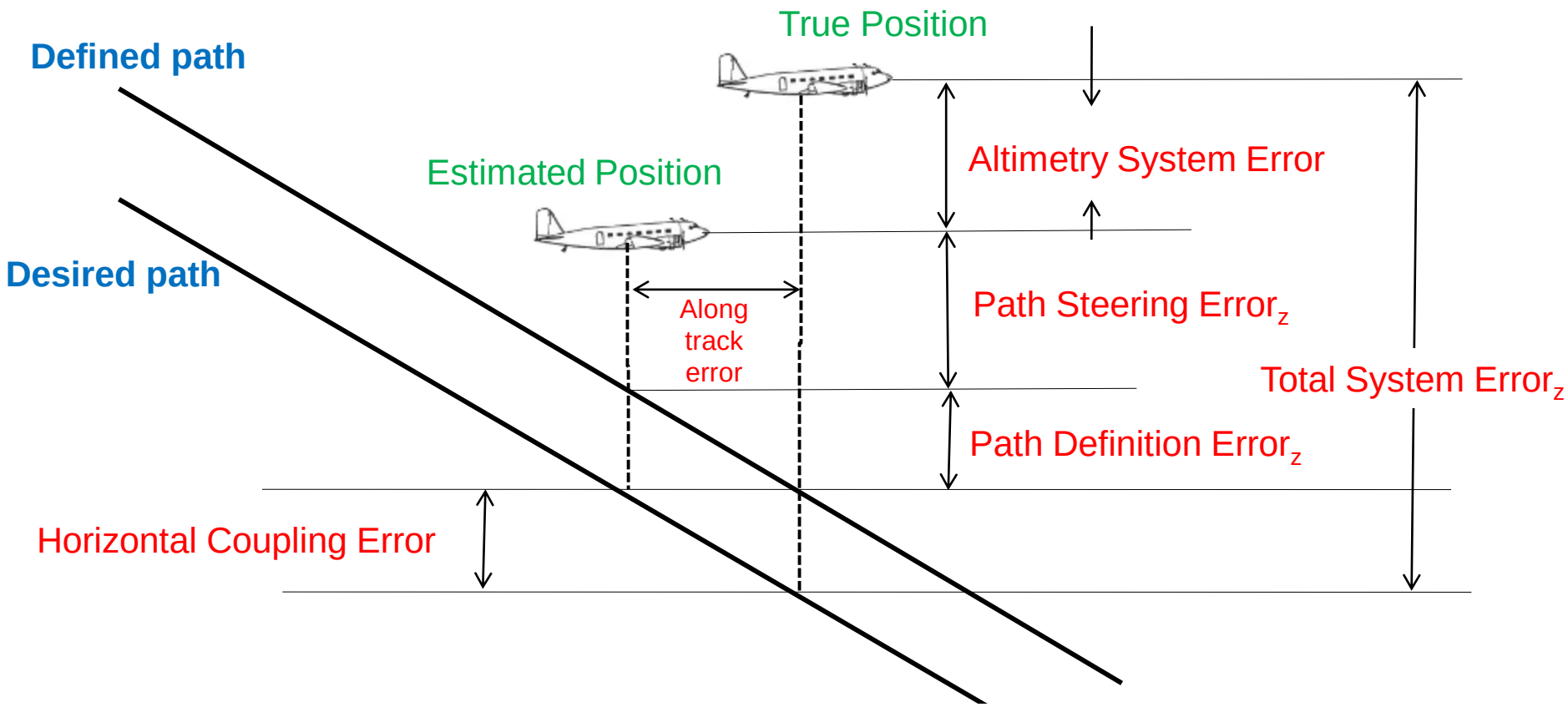
Navigation Error Terms - Lateral



Path Steering Error components: flight technical error and display error

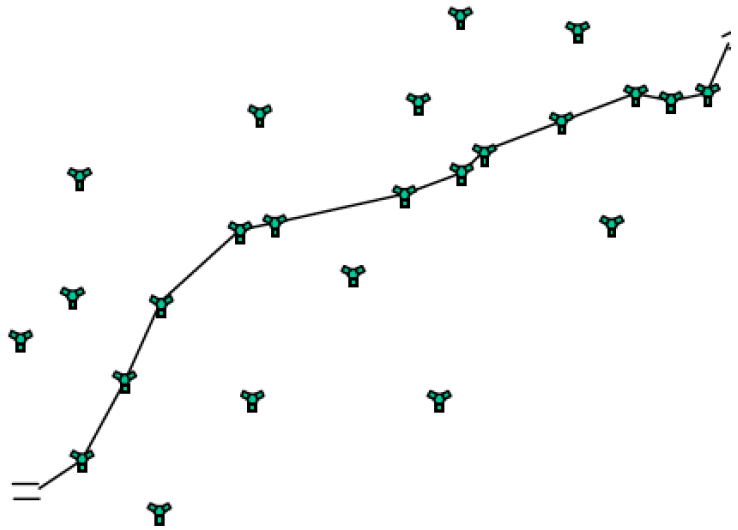
Based on: DO-283A, "Minimum Operational Performance Standards for Required Navigation Performance for Area Navigation," RTCA Inc., 2003

Navigation Error Terms - Vertical

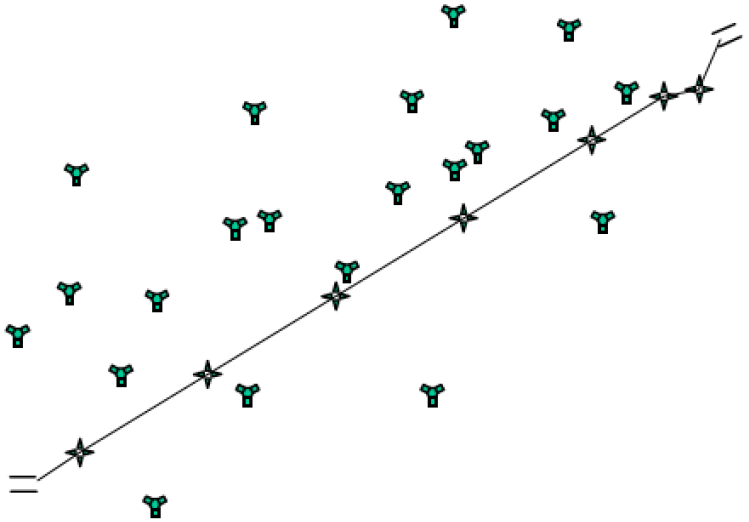


Based on: DO-236B, "Minimum Aviation System Performance Standards: Required Navigation Performance for Area Navigation," RTCA Inc., 2003

Navigation - RNAV



Traditional: routes strongly based on ground navaids



Area Navigation (RNAV)
Fly a trajectory/route through a set of waypoints that do not coincide with ground navaid

Lateral component: LNAV
Vertical component: VNAV

Required Navigation Performance (RNP)

- CONOPS Definition:

A statement of the **navigational performance** necessary for operation within a defined airspace.

Performance-Based Navigation (PBN)

Navigation Applications Without Onboard Monitoring and Alerting (RNAV)

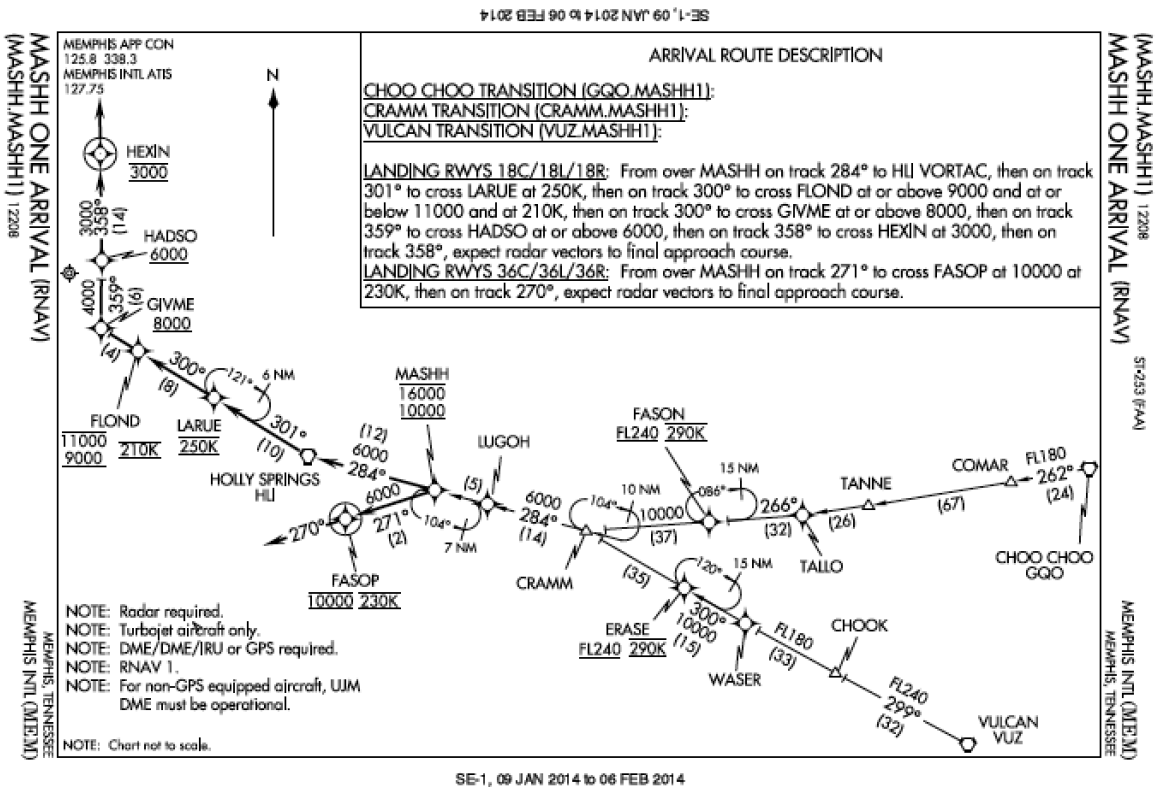
Navigation Applications With Onboard Monitoring and Alerting (RNP)

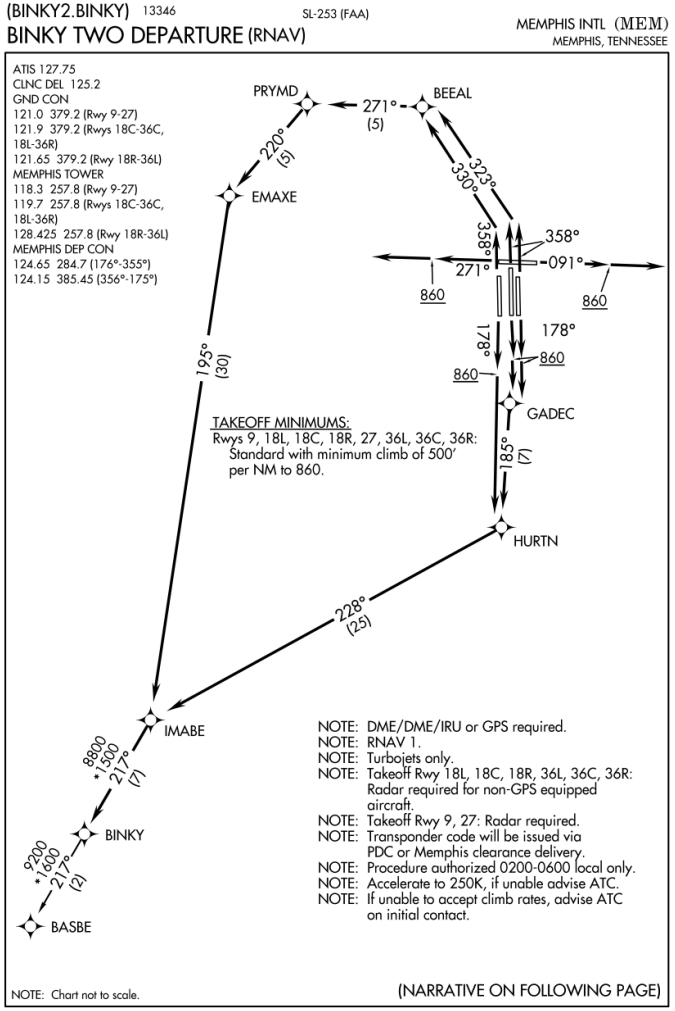
Meet Accuracy, Integrity, Availability and Continuity Requirements

RNAV versus RNP

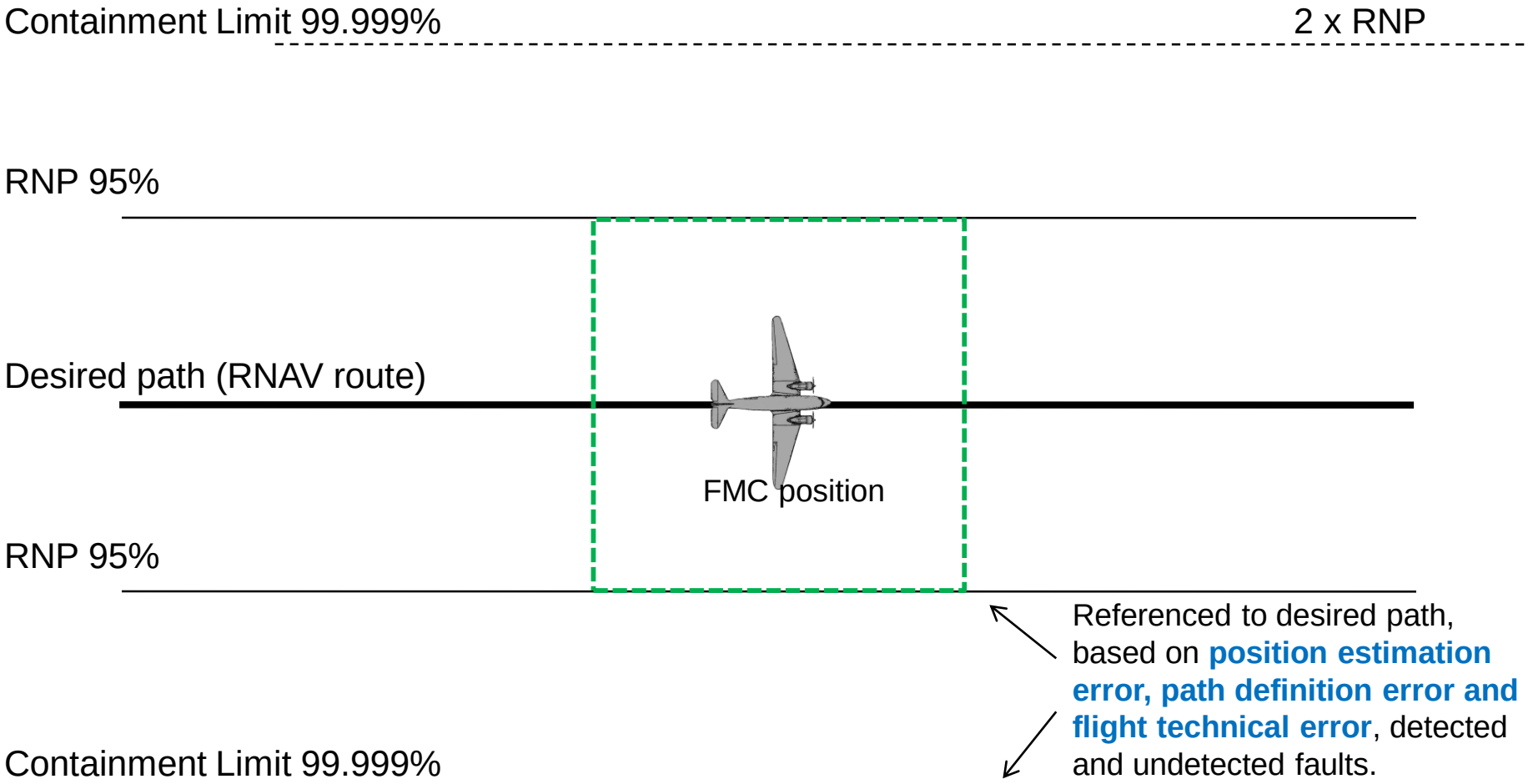
- RNAV
  - Lateral Navigation (LNAV).
    - A function of area navigation (RNAV) equipment which calculates, displays, and provides lateral guidance to a profile or path.
  - Vertical Navigation (VNAV)
    - A function of area navigation (RNAV) equipment which calculates, displays, and provides vertical guidance to a profile or path.
- RNP
  - Required Navigation Performance Level or Type (RNP-X).
    - A value, in nautical miles (NM), from the intended horizontal position within which an aircraft would be at least 95-percent of the total flying time.
  - Required Navigation Performance (RNP) Airspace.
  - A generic term designating airspace, route(s), leg(s), operation(s), or procedure(s) where minimum required navigational performance (RNP) have been established.

Arrival and Departure Routes



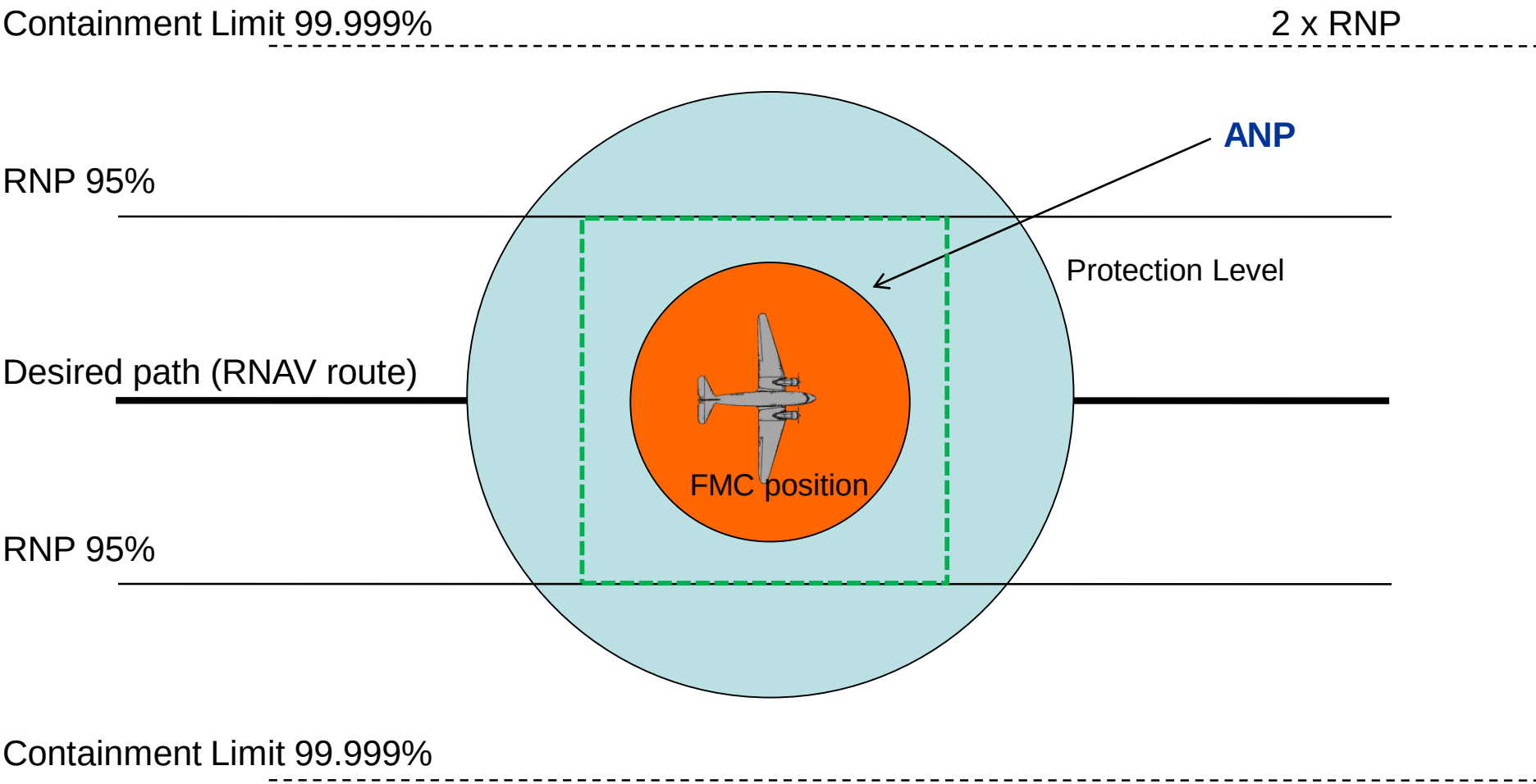


Navigation – Cross-track Containment



See: ICAO Manual on RNP

Navigation – Cross-track Containment



Actual Navigation Performance (ANP):

A measure of the current estimated navigational performance (95% performance). Also referred to as Estimated Position Error (EPE).

Navigation – RNP RNAV Types

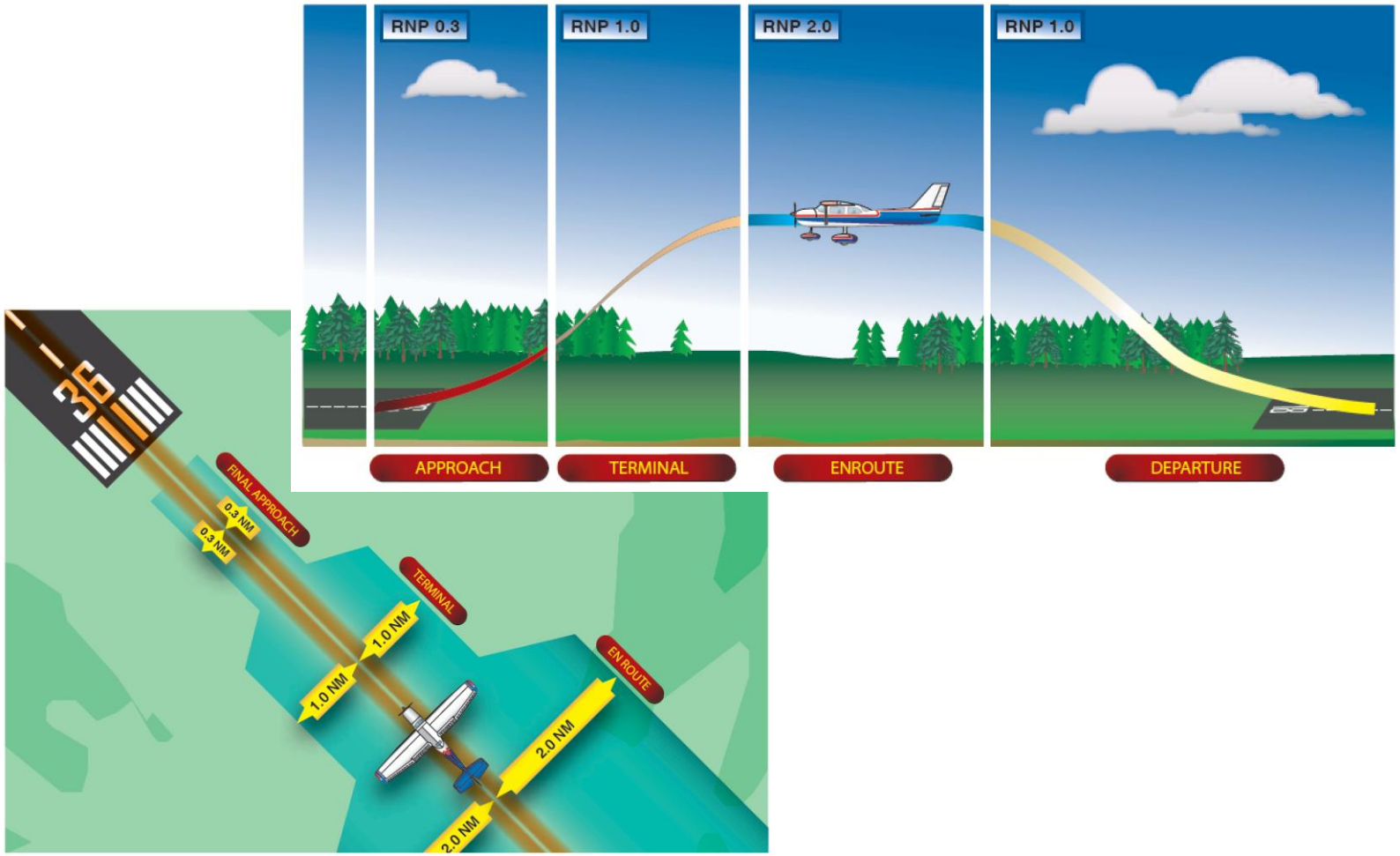
Flight phase	
Oceanic	RNP-4.0 RNAV
En route (domestic)	RNP-2.0 RNAV
Terminal	RNP-1.0 RNAV
Approach	RNP-0.3 RNAV

Assumed Flight Technical Error

Flight phase	Manual (NM)	Coupled (NM)	
		Flight director	Autopilot
Oceanic	2.0	0.5	0.25
En route	1.0	0.5	0.25
Terminal	1.0	0.5	0.25
Approach	0.5	0.25	0.125

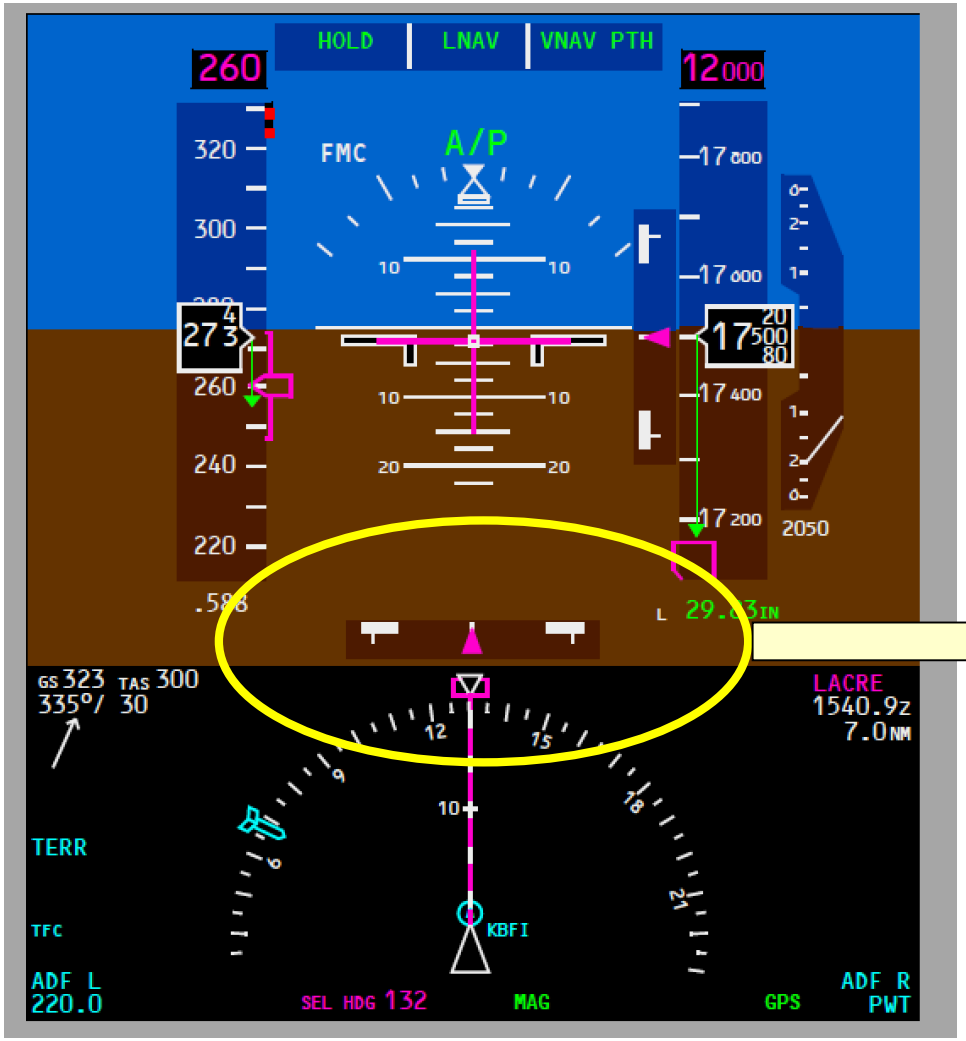
See: ICAO Manual on RNP

Navigation – RNP RNAV Types



From: FAA-H-8083-15B

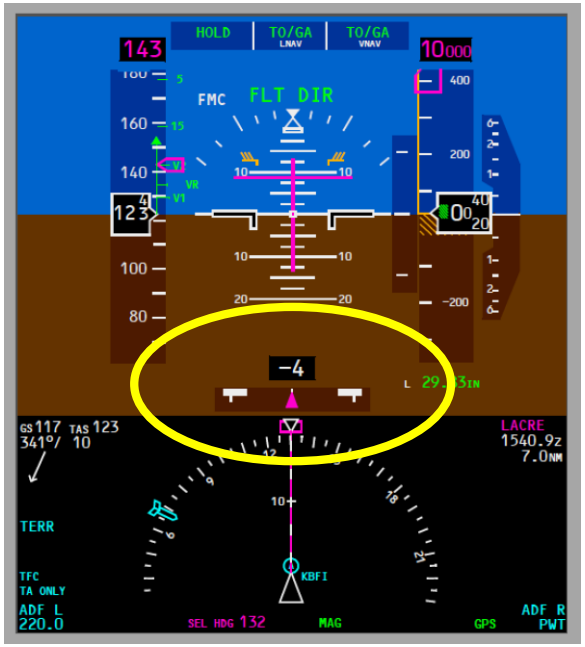
RNP RNAV - Displays

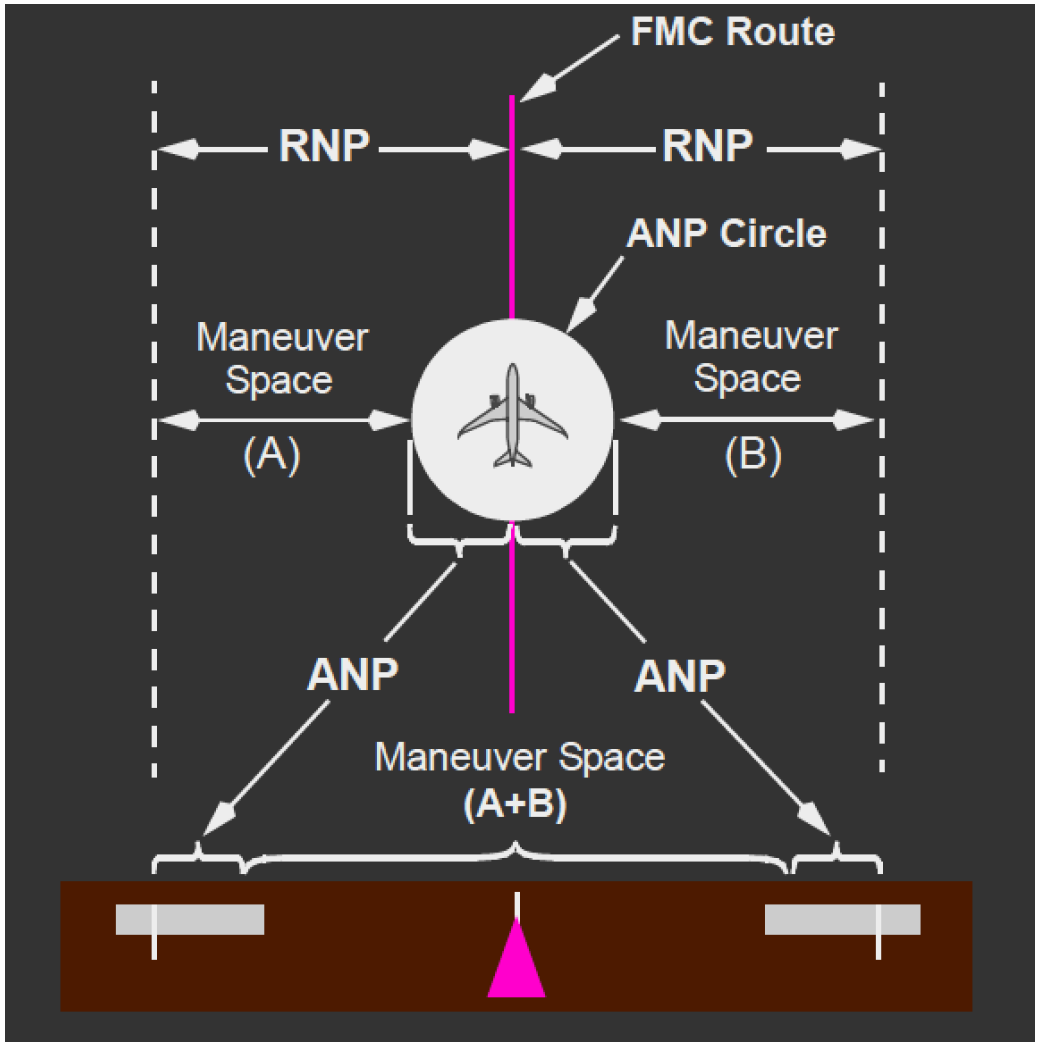


RNP RNAV:
Lateral component
(Lateral navigation, LNAV)

From: B787 FCOM

RNP RNAV – Display Concept





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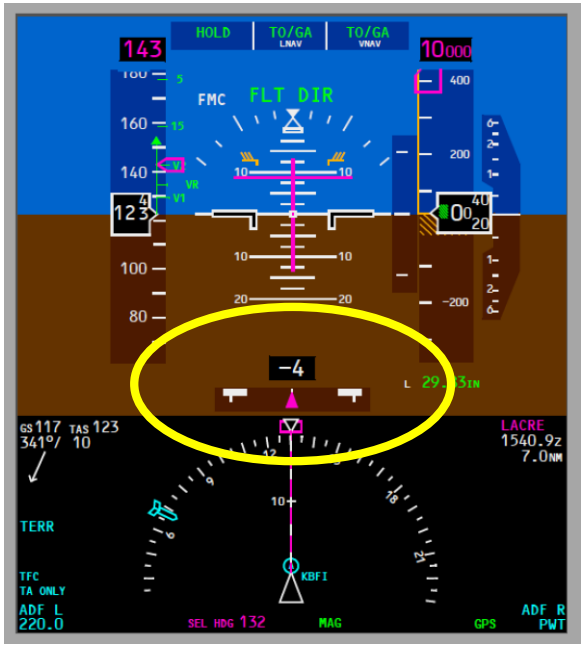
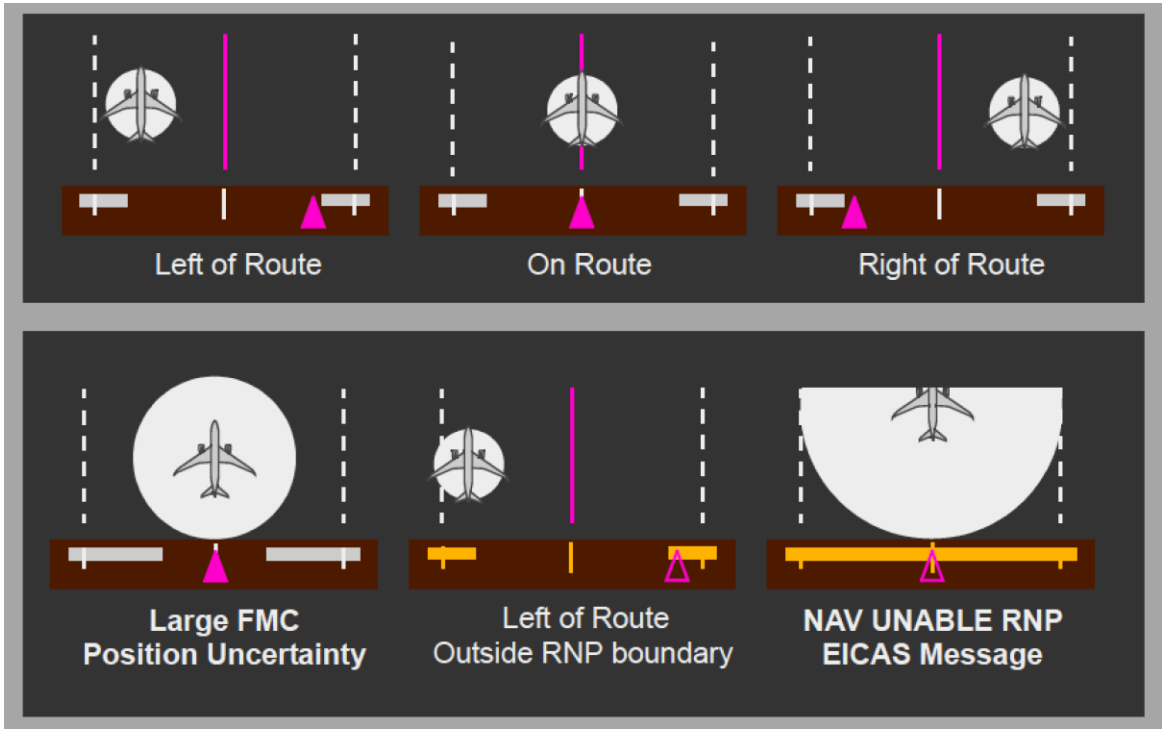
RNP RNAV – Display Concept





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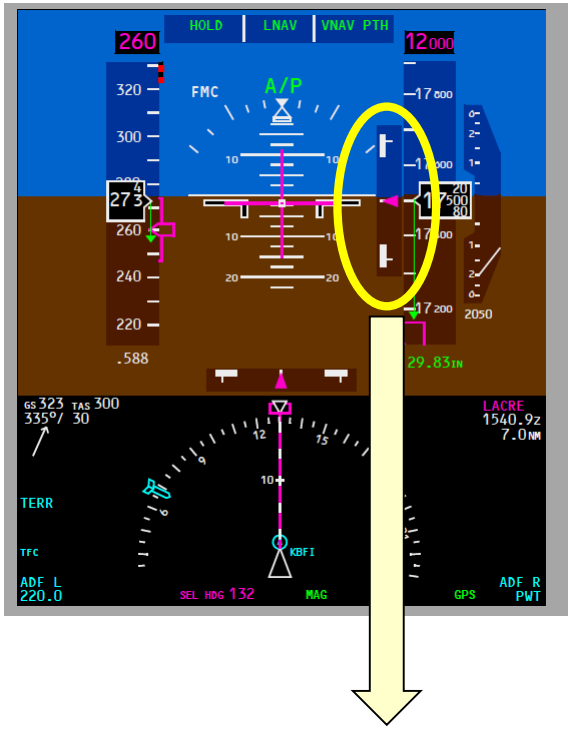
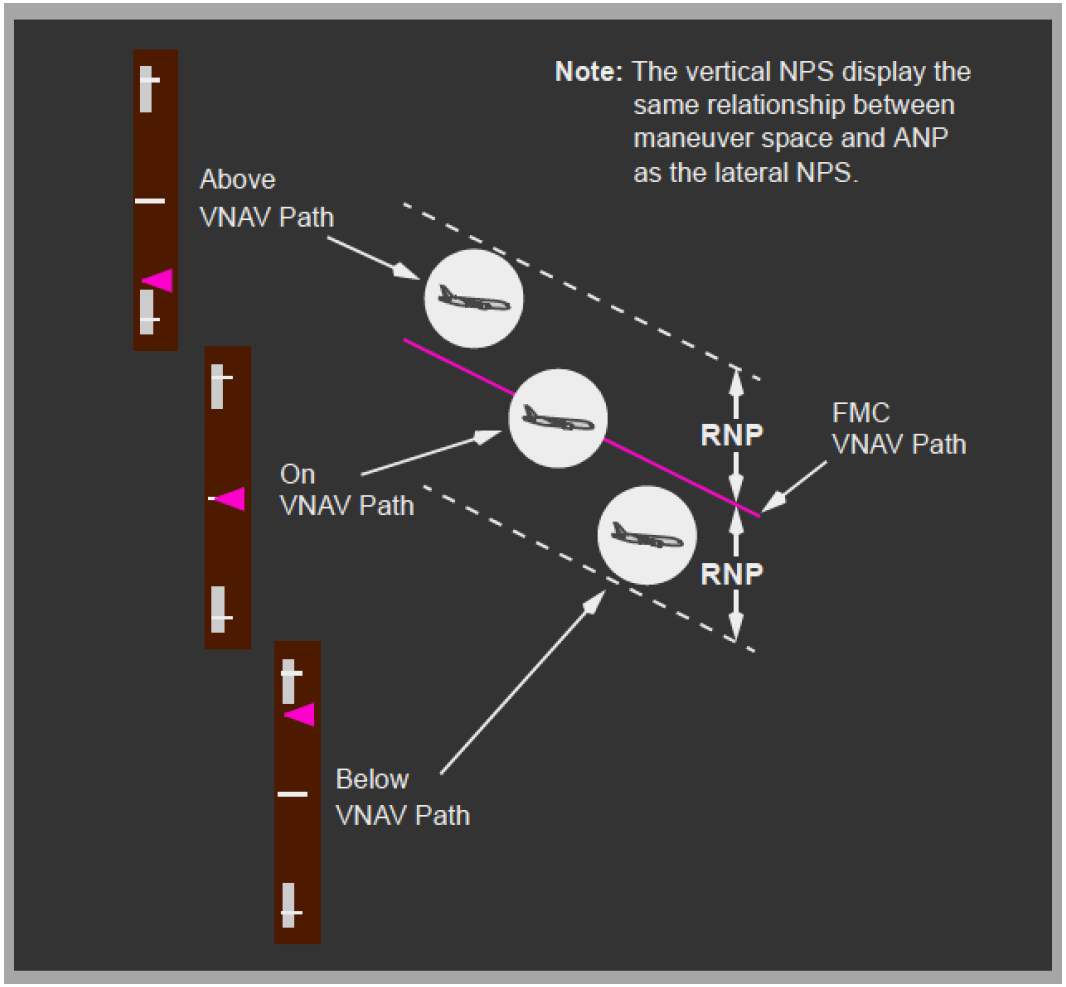
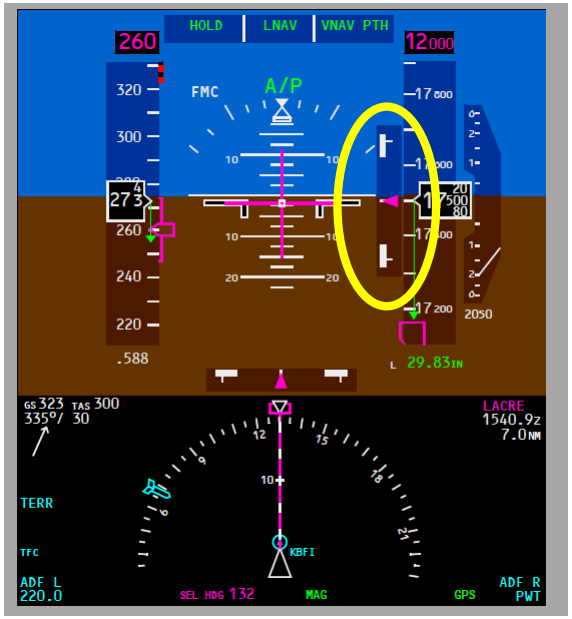
RNP RNAV – Display Concept

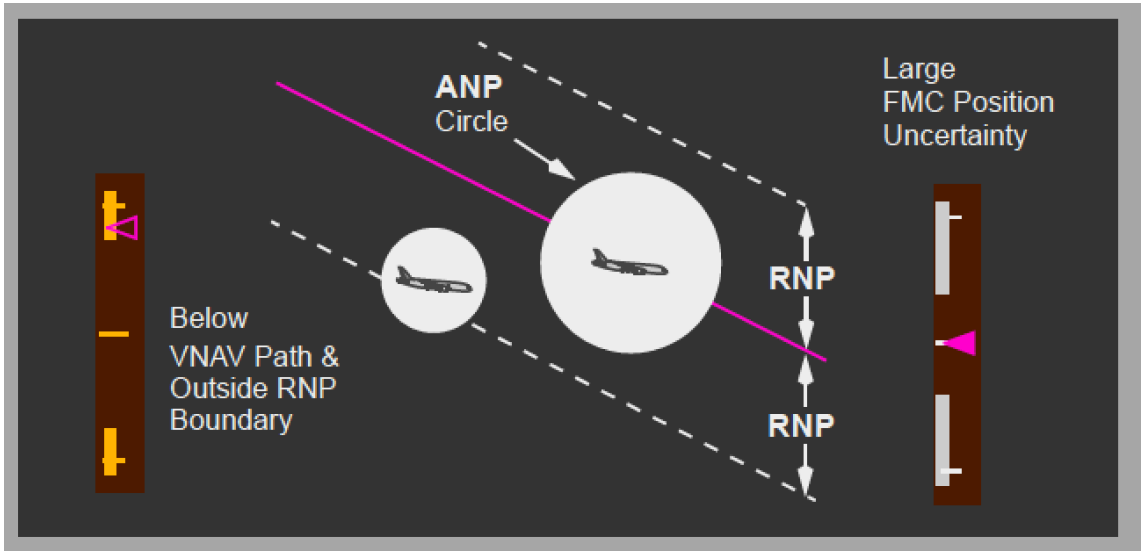


RNP RNAV:
Vertical component
(vertical navigation, VNAV)



RNP RNAV – Display Concept





Approach Phase of Flight

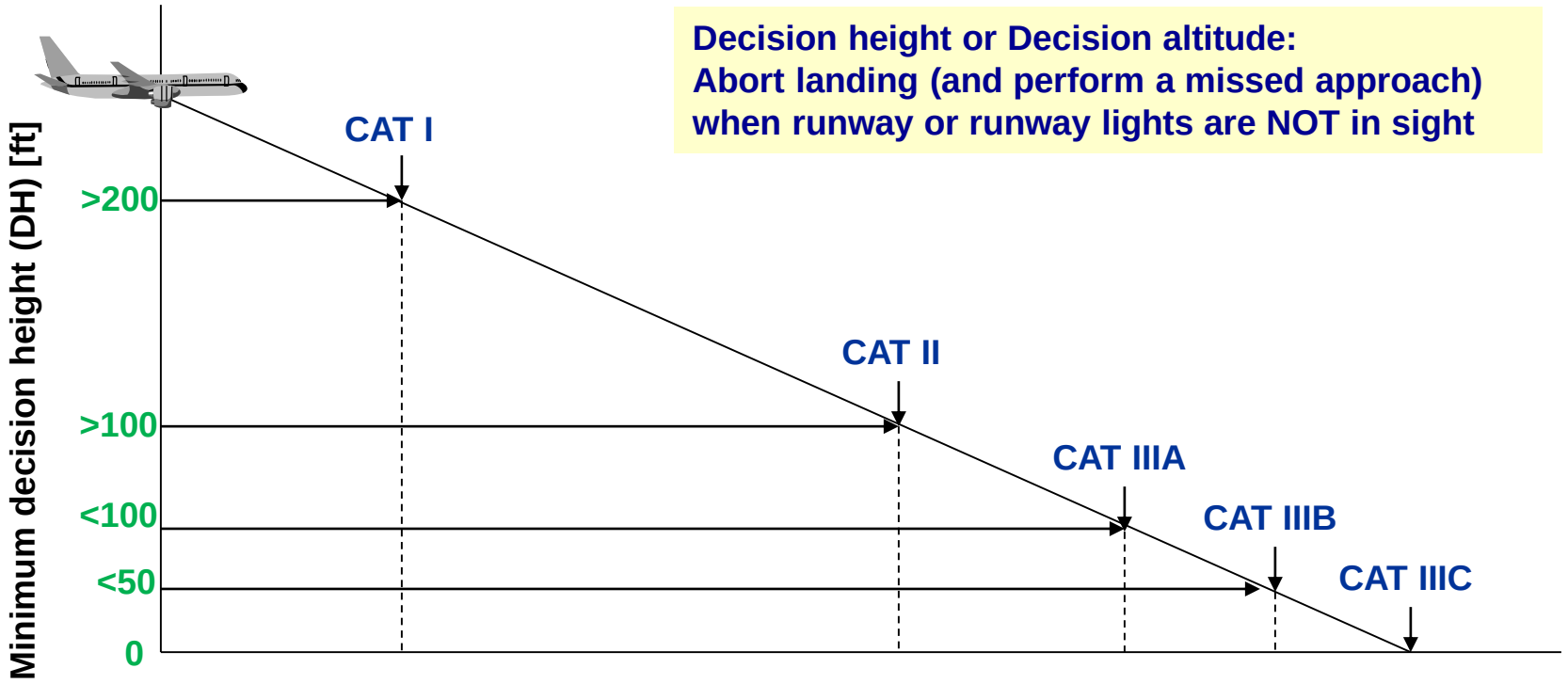
- Visual Flight Rules: VFR
  - Ceiling > 1000 ft
  - Horizontal visibility > 3 NM
- Instrument Flight Rules: IFR
  - **Non-precision**: lateral guidance only
  - **Precision**: lateral and vertical guidance
    - Category I,II,IIIA-B-C

Landing Minimums

- **Precision Approach**
  - Decision Height (DH): height above threshold (HAT) at which a decision must be made during an instrument landing system (ILS), microwave landing system (MLS), or precision approach radar (PAR) instrument approach to either continue the approach or to execute a missed approach.
  - or Decision Altitude (DA)
- **Non-precision Approach**
  - Minimum Descent Altitude (MDA): the lowest altitude, expressed in feet above mean sea level (MSL), to which descent is authorized on final approach, or during circle-to-land maneuvering, in execution of a standard instrument approach procedure (SIAP) where no electronic glideslope is provided
  - MDA/H differs from a DA/H in that the aircraft must be flown in such a way that it does not descend below the MDA/H unless the required visual reference has been established.

See: Aeronautical Information Manual

Geometry of the Precision Approach



- Instrument Landing System (ILS)
- Ground Based Augmentation System (GBAS)

Categories of Service

	Performance Category I	Performance Category II	Performance Category IIIA	Performance Category IIIB	Performance Category IIIC
Decision Height	>200ft 30m	>100ft 15m	<100ft 15m	<50ft	N/A
Runway Visual Range (RVR)	≥1,800ft	≥1,200ft	≥700ft	≥150ftm	N/A
Comments	Majority of all approaches				Most stringent

- Category of Approach
  - Depends on weather minimum
  - Cloud Ceiling
  - Performing the approach depends upon aircraft and ground equipage and certification

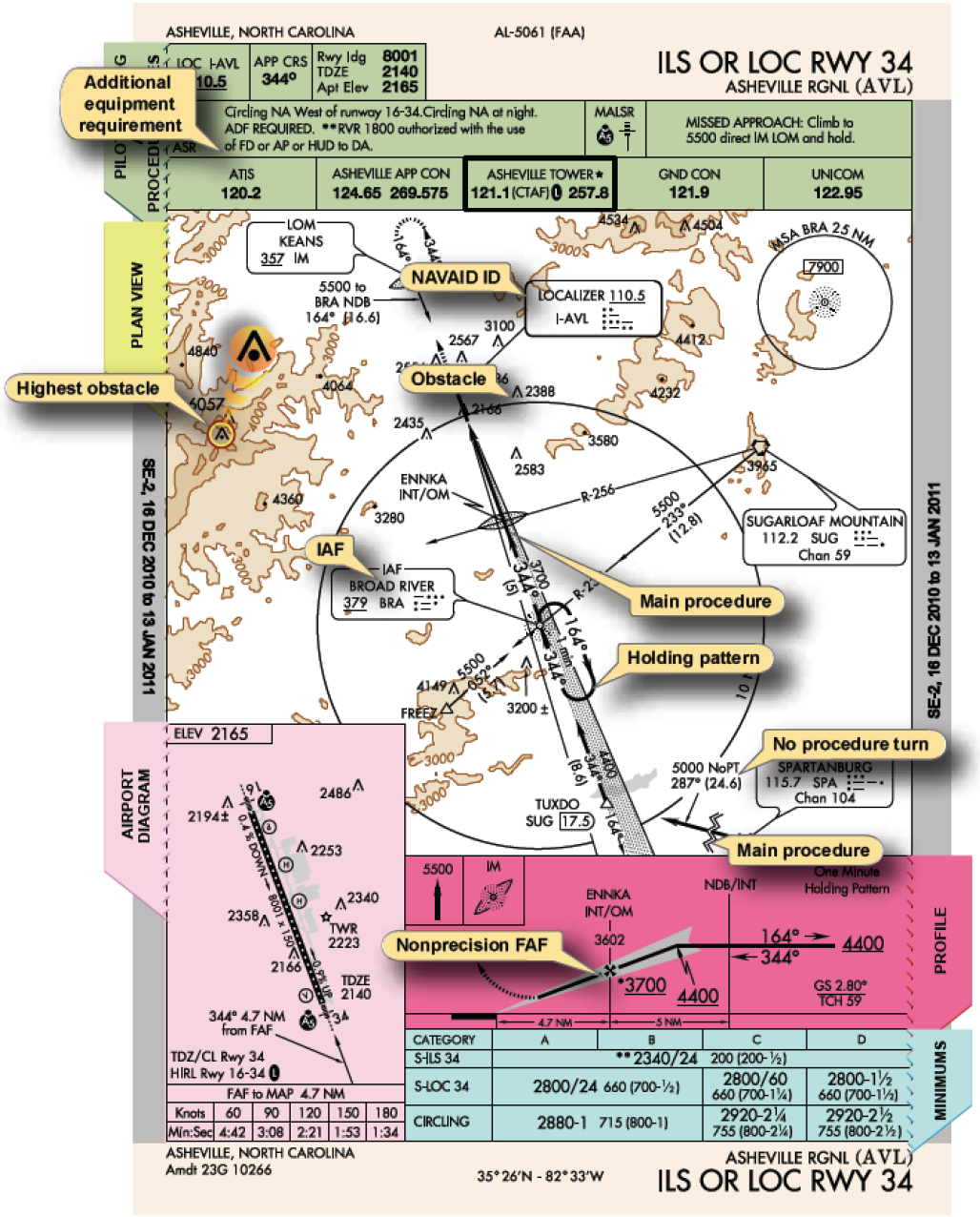
See: FAA-H-8261-1A

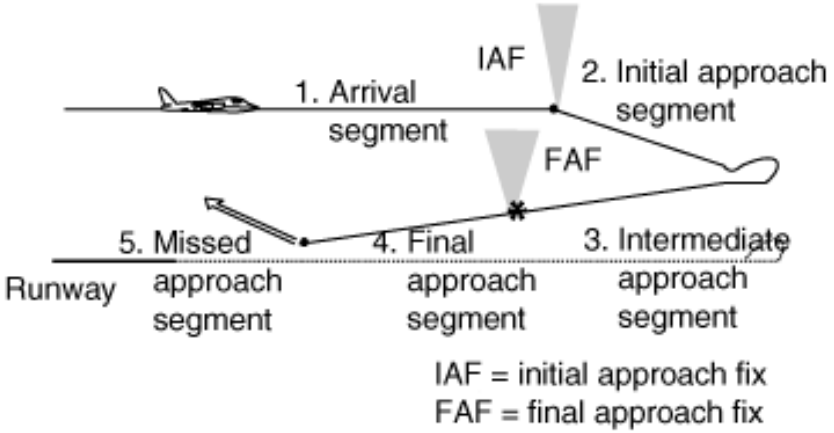
Space-Based Augmentation System* (SBAS)

- **GPS NPA (LNAV)**
  - Non-Precision Approach (NPA) procedure which uses GPS and/or WAAS for Lateral Navigation (LNAV)
- **LNAV/VNAV (Lateral Navigation / Vertical Navigation)**
  - Approaches using lateral guidance (556m lateral limit) from GPS and/or WAAS and vertical guidance provided by either the barometric altimeter or WAAS.
- **LPV (Localizer Performance with Vertical guidance**)
  - Like LNAV/VNAV but more precise (40m lateral limit),
  - Enables descent to 200-250 feet above the runway, and
  - Can only be flown with a WAAS receiver.
- **LP (Localizer Performance)** is a future NPA procedure that uses the high precision of LPV for lateral guidance and barometric altimeter for vertical guidance

**in USA: Wide Area Augmentation System (WAAS)*

Approach Plates





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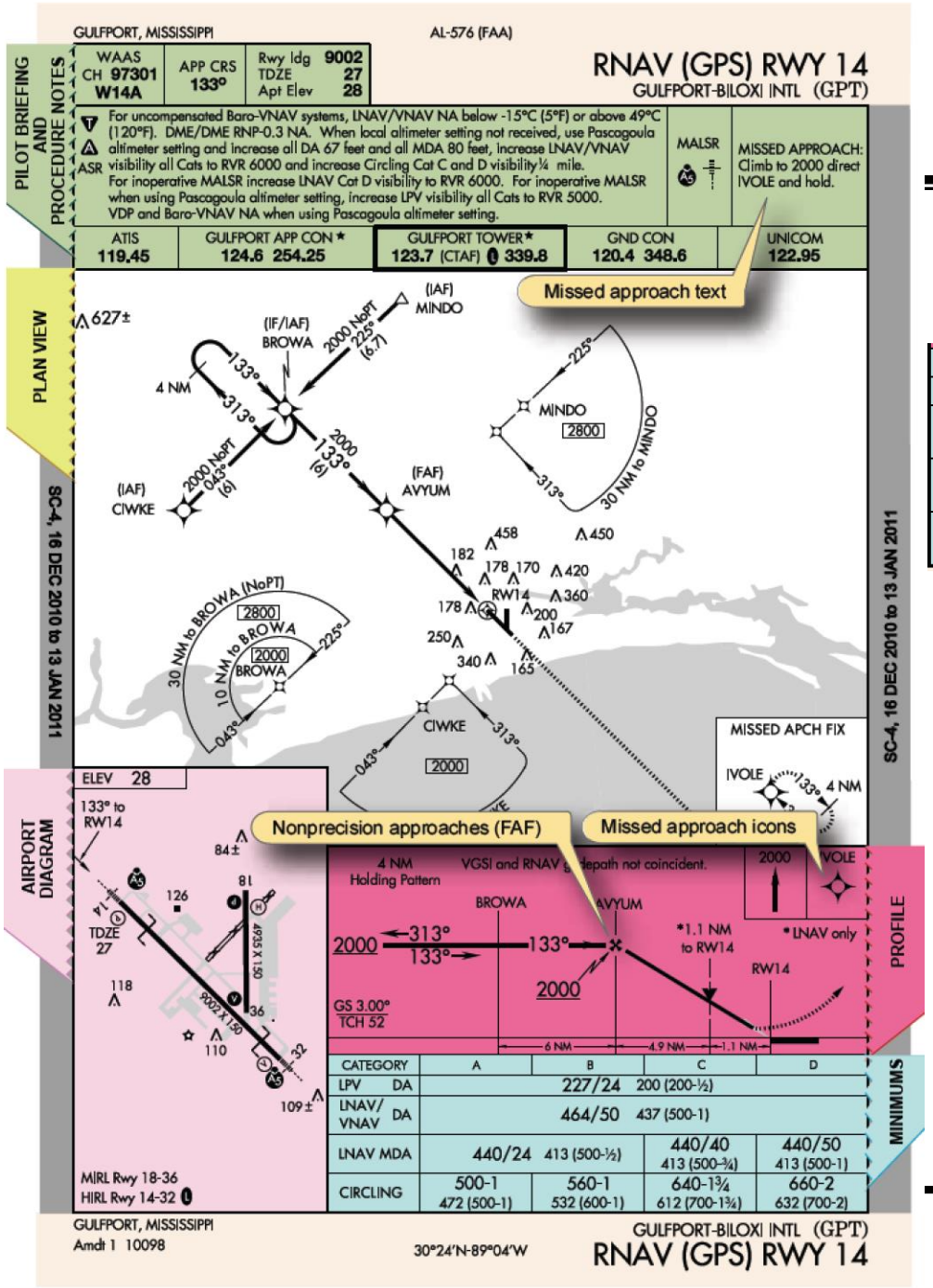


Aircraft Approach categories
(see next slide)

CATEGORY	A	B	C	D
LPV DA	227/24	200 (200-½)		
LNAV/VNAV DA	464/50	437 (500-1)		
LNAV MDA	440/24	413 (500-½)	440/40 413 (500-¾)	440/50 413 (500-1)
CIRCLING	500-1 472 (500-1)	560-1 532 (600-1)	640-1¾ 612 (700-1¾)	660-2 632 (700-2)

DA/RVR
Decision Altitude/Runway Visual Range

HAT
(Height Above Threshold)

CATEGORY	A	B	C	D
LPV DA	227/24	200 (200-½)		
LNAV/VNAV DA	464/50	437 (500-1)		
LNAV MDA	440/24	413 (500-½)	440/40 413 (500-¾)	440/50 413 (500-1)
CIRCLING	560-1 532 (600-1)		640-1¾ 612 (700-1¾)	660-2 632 (700-2)

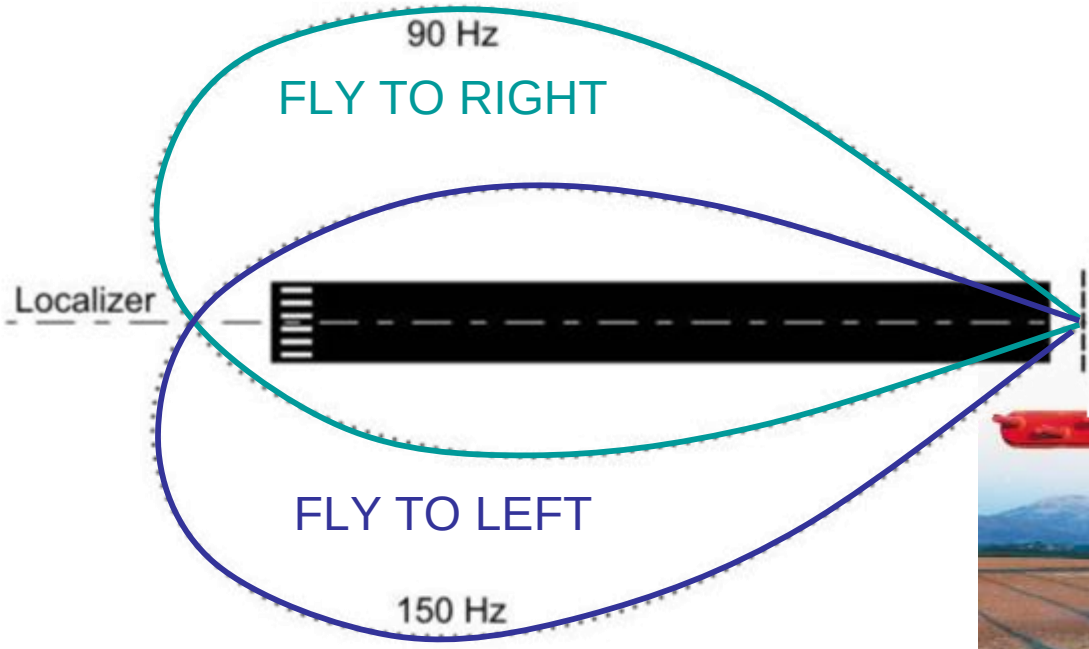
MDA/RVR
Minimum Descent Altitude/Runway Visual Range

Aircraft Approach Categories

Aircraft category	1.3*stall speed	Range of IA speeds	Range of FA speeds	Max circling speeds	Max intermediate MA speeds	Max final MA speeds	Typical Aircraft
A	<91	90-150	70-110	100	100	110	Small single engine
B	91-120	120-180	85-130	135	130	150	Small multi engine
C	121-140	160-240	115-160	180	160	240	Airline jet
D	141-165	185-250	130-185	205	185	265	Large jet/military jet
E	166-210	185-250	155-230	240	230	275	Special military

IA = Initial Approach
FA = Final Approach
MA = Missed Approach

ILS: Localizer



Localizer
(Lateral Guidance - VHF Frequencies)

Provides lateral guidance: guides the plane along the center line of the runway
Frequency: 108-112 MHz
Range: 25 NM at 10˚ , 17 NM at 35˚
Antenna located at the end of runway, on the center line

ILS: Glideslope

Glide slope:

Provides vertical guidance: guides plane with descent slope of ≈3˚ to the runway

Frequency: 329.3-335.0 MHz

Antenna located besides runway near threshold

ILS: Difference in Depth of Modulation

Total signal modulation: $$M = \frac{A_{150Hz} + A_{90Hz}}{A_{carrier}}$$

(A=received signal amplitude)

Difference in Depth of Modulation $$DDM = \frac{A_{150Hz} - A_{90Hz}}{A_{carrier}}$$

90 Hz dominant: DDM negative → Fly right
At centerline: DDM zero → on course
150 Hz dominant: DDM positive → Fly right

Full scale LOC = 0.155 DDM = 0.5xwidth = 350' at threshold = 2 dots = 150uA

Full scale CatI GS = 0.175DDM = 0.14xGPA = 2 dots = 150uA = 0.82 deg.

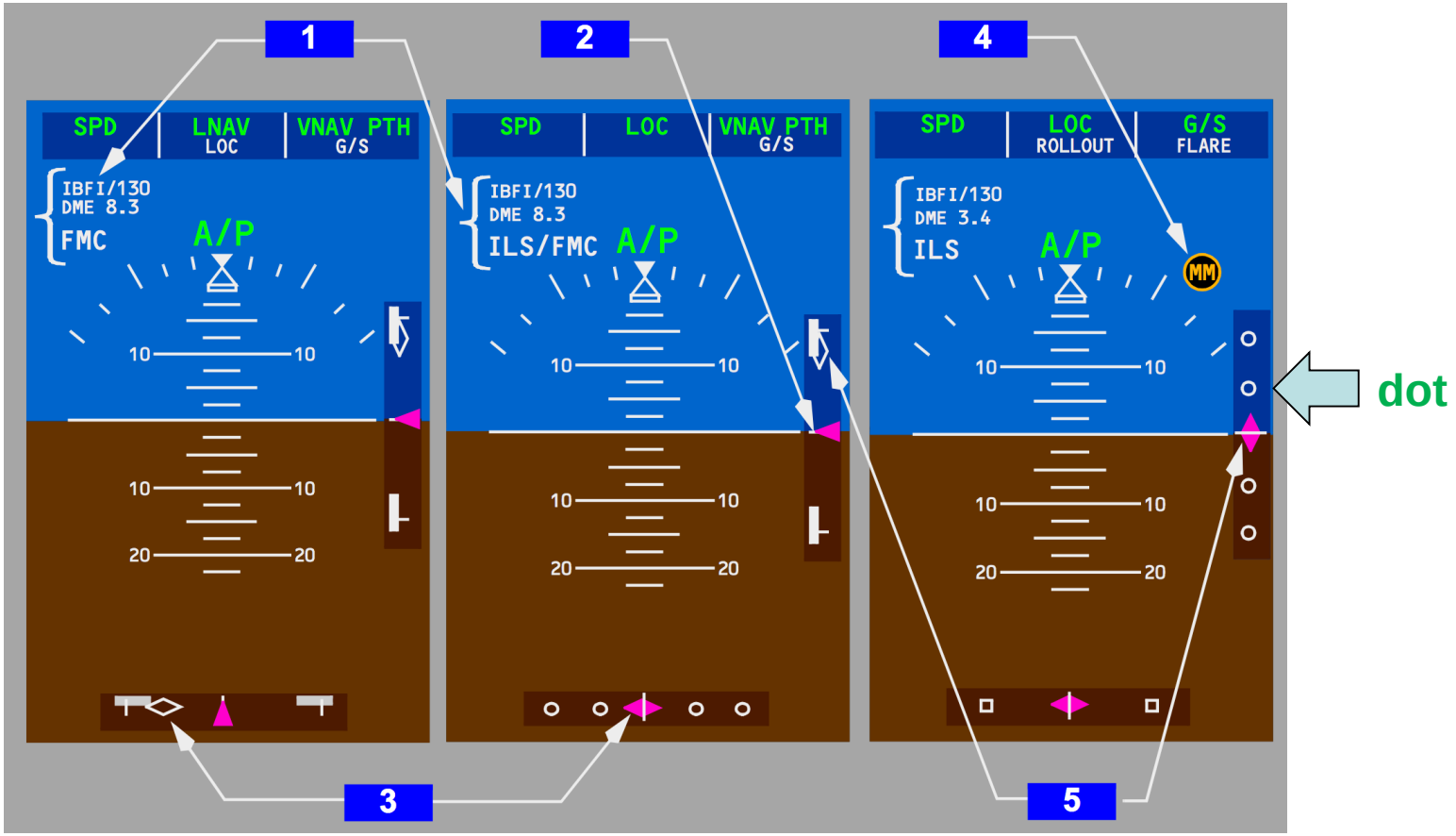
Full scale Cat II/III GS = 0.175 DDM = 0.12xGPA = 2 dots = 150uA = 0.7 deg

Cross-pointer Course Deviation Indicator (CDI)

ILS Tone Ratio/DDM Conversion Chart

TONE	LOCALIZER (LOC)			GLIDESLOPE (GS)		
Ratio (dB)	*DDM	% Mod (Larger/Smaller)	RCVR Deflection (uA)	*DDM	% Mod (Larger/Smaller)	RCVR Deflection (uA)
0.0	0.000	20.00/20.00	0	0.000	40.00/40.00	0
0.5	0.012	20.58/19.42	12	0.023	41.15/38.85	20
0.98	0.023	21.13/18.87	22	0.045	42.25/37.75	39
1.98	0.0456	22.28/17.72	44	0.091	44.55/35.45	78
2.00	0.0458	22.29/17.71	44	0.092	44.59/35.41	79
2.01	0.046	22.30/17.70	45	0.092	44.60/35.40	79
3.86	0.087	24.37/15.63	84	0.175	48.75/31.25	150
4.00	0.091	24.53/15.47	88	0.181	49.05/30.95	155
4.11	0.093	24.65/15.35	90	0.186	49.30/30.70	159
7.10	0.155	27.75/12.25	150	0.310	55.50/24.50	266
7.50	0.163	28.14/11.86	158	0.325	56.27/23.73	279
9.54	0.200	30.00/10.00	194	0.400	60.00/20.00	343
∞	0.400	40.00/00.00	387	0.800	80.00/00.00	686

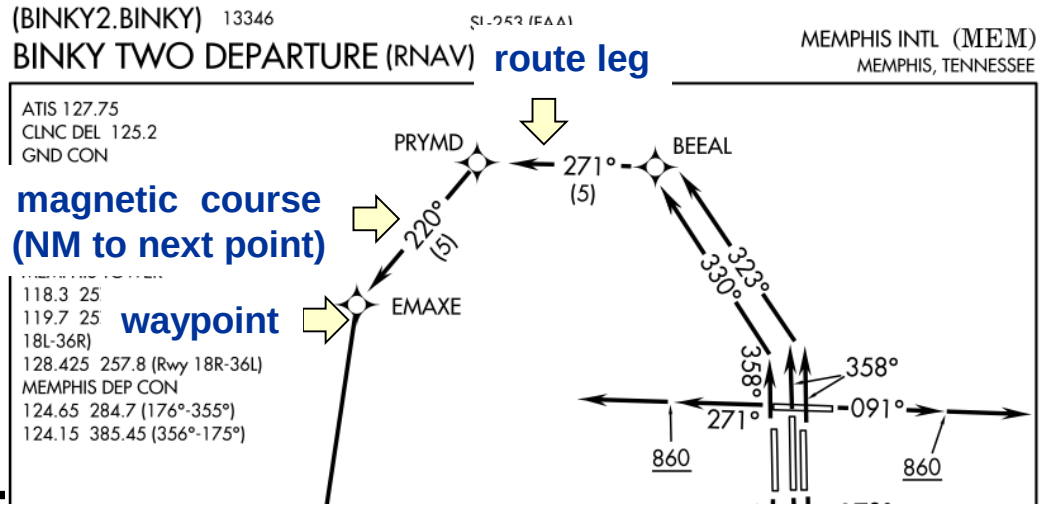
Instrument Landing System Indications



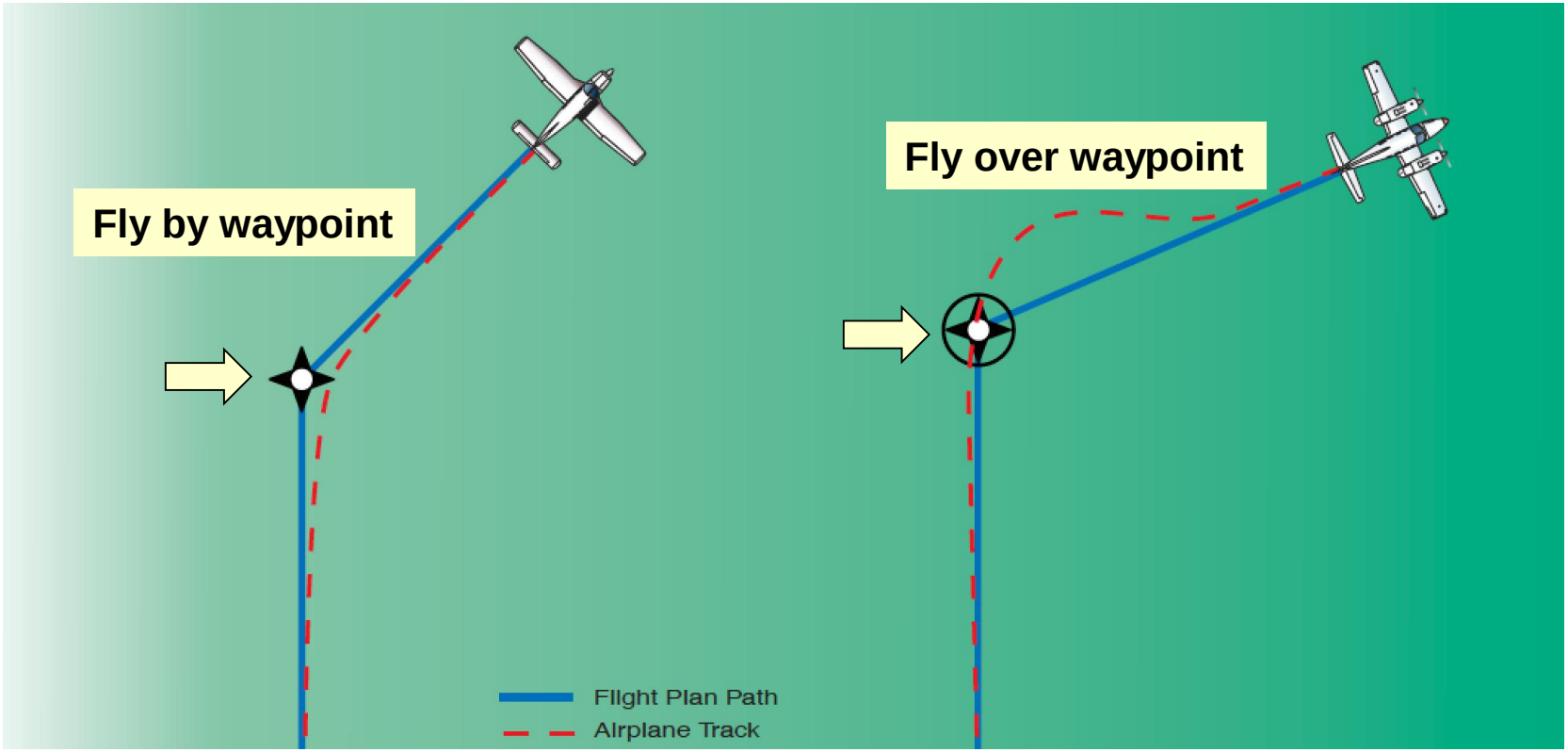
See: B787 FCOM pp. 850

Flight Planning: Routes

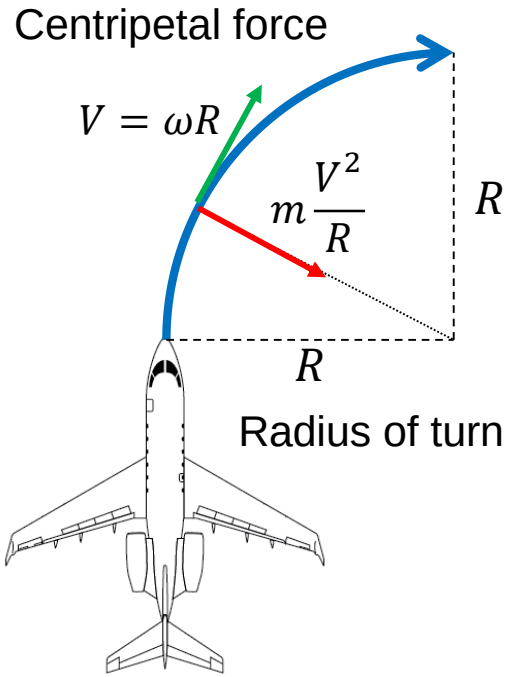
- Desired paths:
  - Routes (e.g. airways, oceanic tracks)
  - User preferred trajectories/company routes
  - Standard instrument departure (SID) procedures
  - Standard Terminal Arrival (STAR) procedures
  - Instrument approach procedures

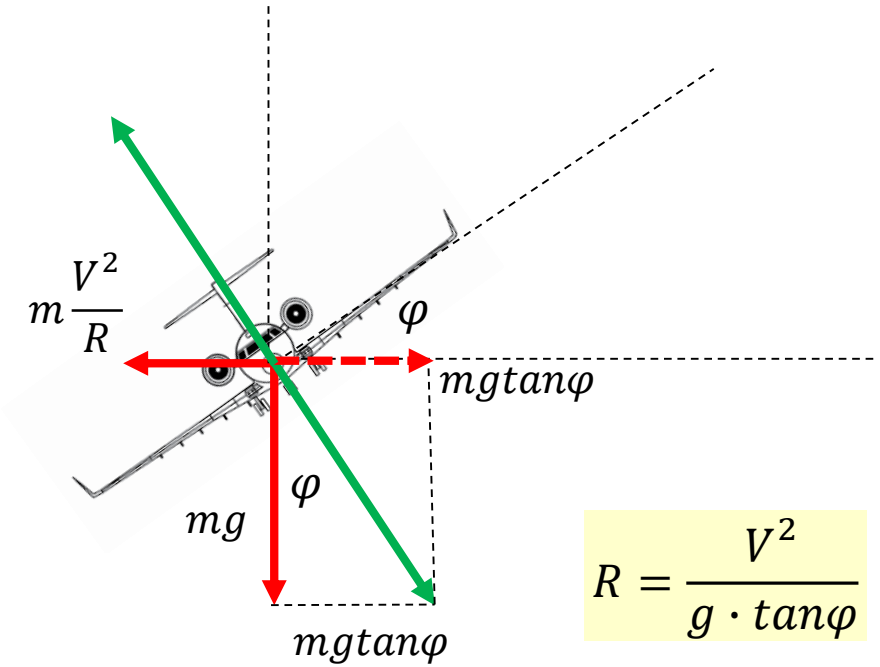


Waypoint Types



Aircraft Bank Maneuver



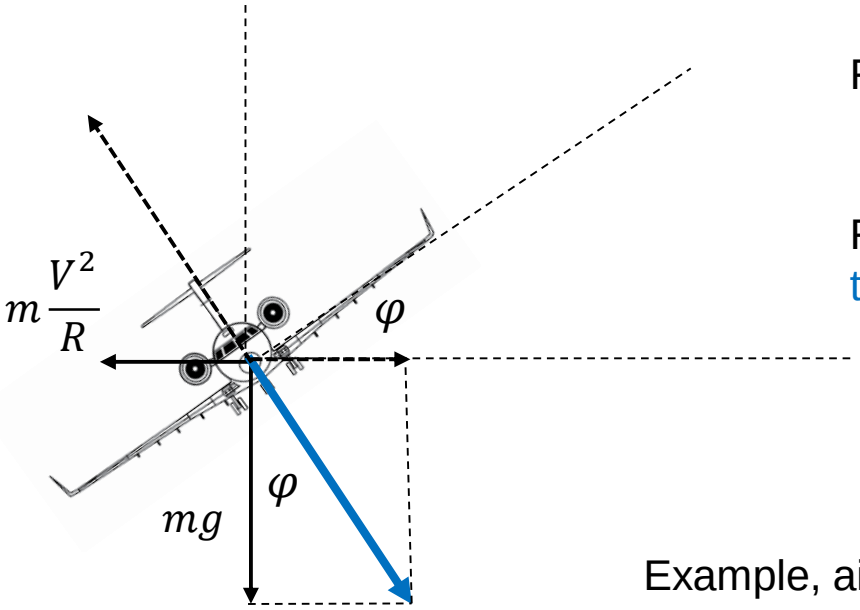


$$R = \frac{V^2}{g \cdot tan\varphi}$$

$$F = m\frac{V^2}{R} = m\omega^2 R$$

$$V = \omega R$$

Aircraft Bank Maneuver



$$F = \frac{mg}{cos\varphi} \Rightarrow a = \frac{g}{cos\varphi}$$

For example, in a $60°$ turn: $a = \frac{g}{0.5} = 2g$; the 2g turn

For light aircraft a standard-rate-turn or rate one turn (ROT) is defined as a $3°/second$ turn.

Perform a $360°$ turn in 2 minutes.

Example, aircraft at a speed to 150kts performing a ROT:

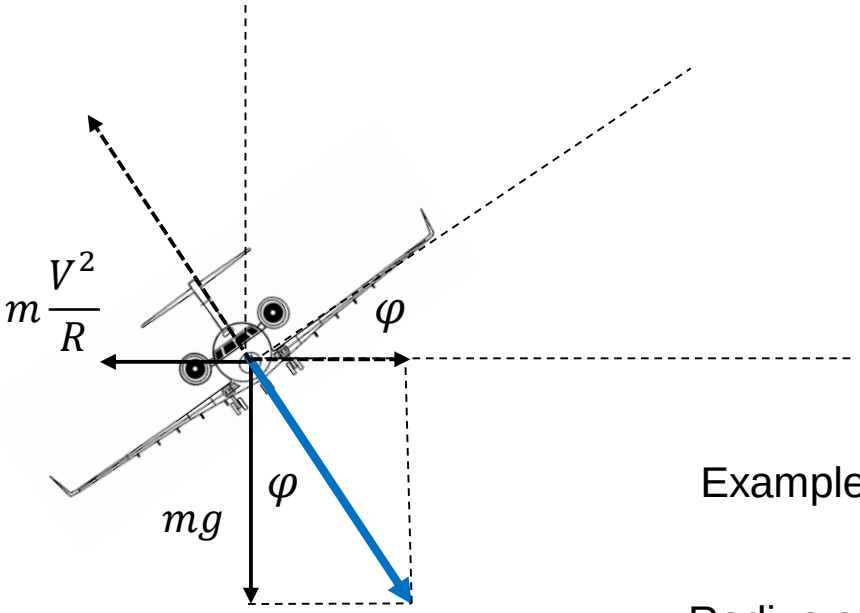
Radius of turn: $R = \frac{v}{\omega} = \frac{77}{(\frac{3\pi}{180})} \approx 1{,}474m$

Bank angle: $\varphi = \tan^{-1}\frac{V^2}{gR} = \tan^{-1}\frac{V\omega}{g} = \tan^{-1}\frac{(77)(\frac{3\pi}{180})}{9.81} \approx 22.4°$

Acceleration: $a = \frac{g}{cos\varphi} \approx 1.08g$

Close to '1g'

Aircraft Bank Maneuver



$$F = \frac{mg}{cos\varphi} \Rightarrow a = \frac{g}{cos\varphi}$$

For heavy aircraft we use a rate half turn is defined as a $1.5°/second$ turn.

Perform a $360°$ turn in 4 minutes.

Example, aircraft at a speed to 250kts performing a RHT:

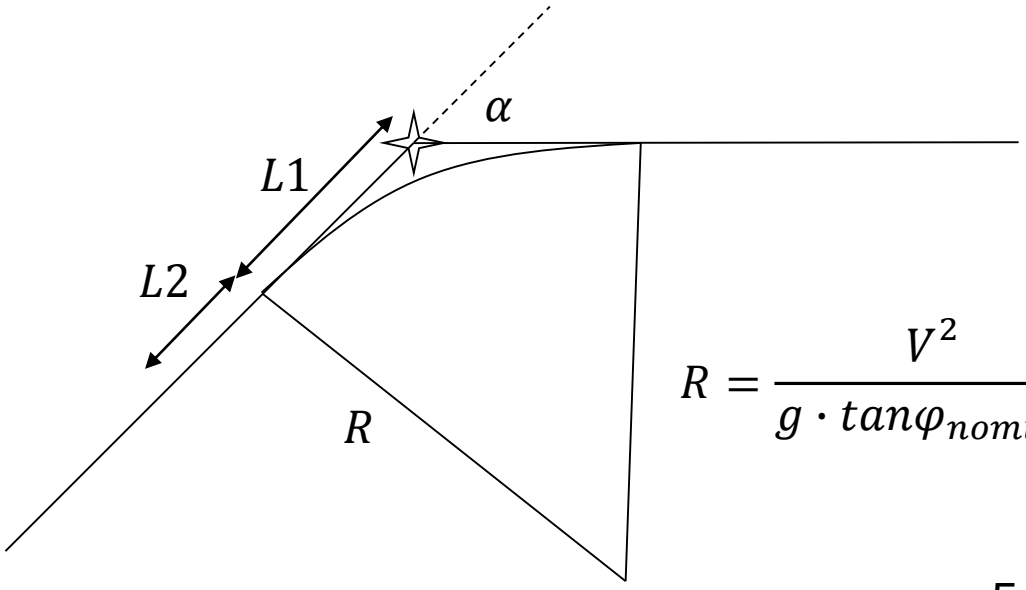
Radius of turn: $R = \frac{v}{\omega} = \frac{129}{(\frac{1.5\pi}{180})} \approx 4{,}913m$

Bank angle: $\varphi = \tan^{-1}\frac{V^2}{gR} = \tan^{-1}\frac{V\omega}{g} = \tan^{-1}\frac{(129)(\frac{1.5\pi}{180})}{9.81} \approx 18.9°$

Acceleration: $a = \frac{g}{cos\varphi} \approx 1.06g$

Close to '1g'

Fly-by Waypoint



$$R = \frac{V^2}{g \cdot tan\varphi_{nominal}}$$

Track/course change: α

Turn initiation distance: L

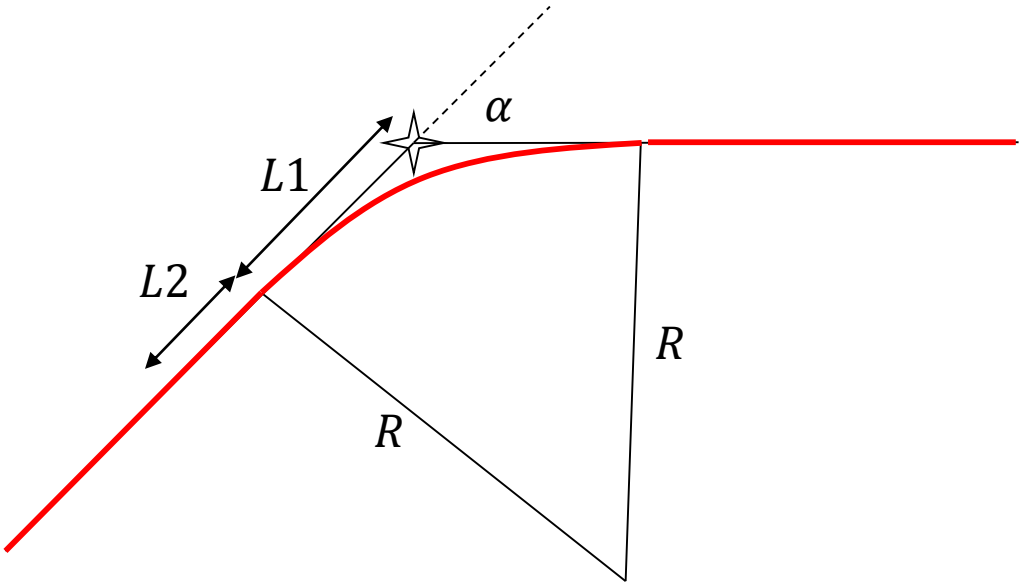
$$L = \underbrace{R \cdot tan(0.5\alpha)}_{L1} + \underbrace{roll\ in\ distance}_{L2}$$

5 second bank establishment time

$$L2 = cV$$

speed in m/s

Fly By Waypoint



Track/course change: α

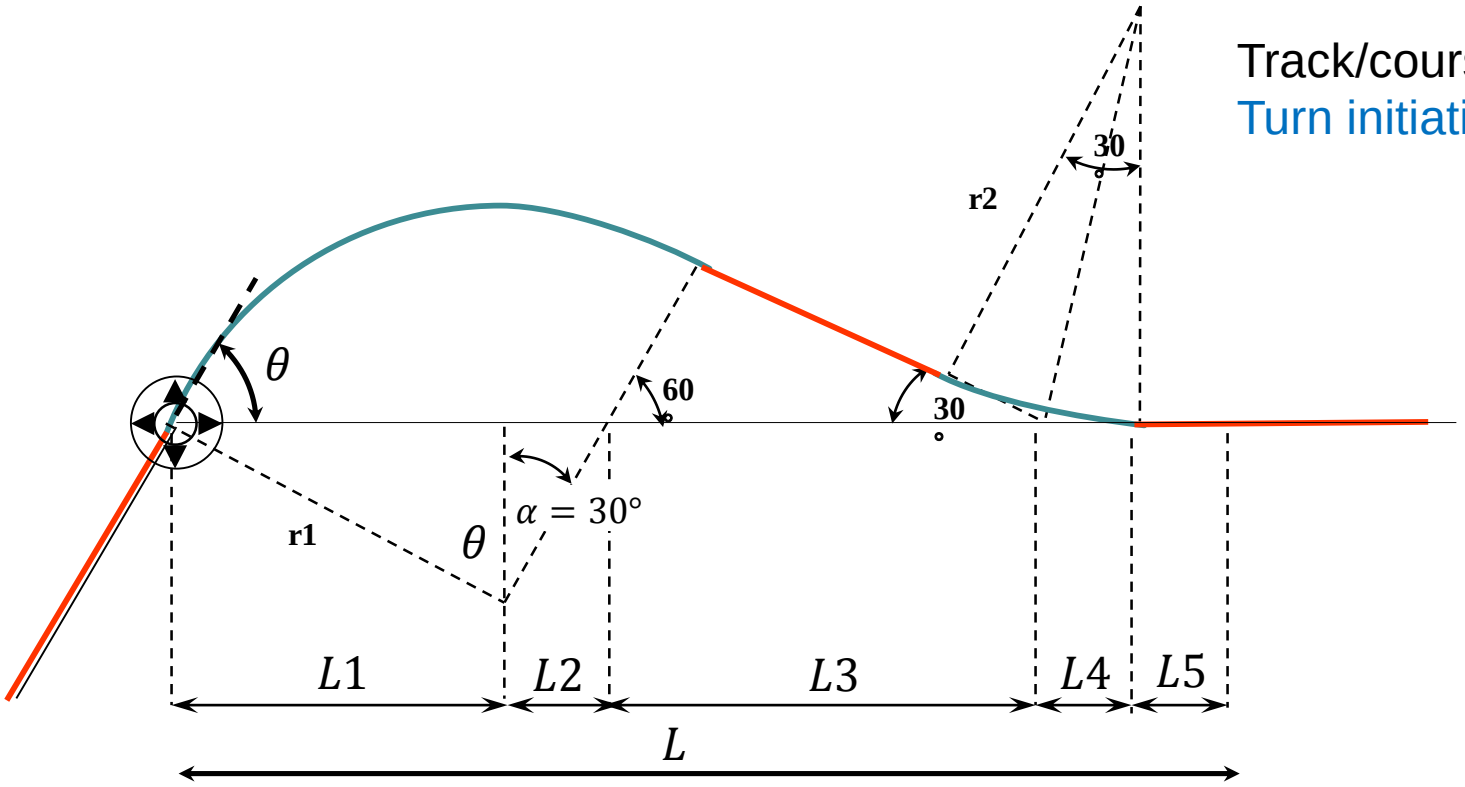
Turn initiation distance: L

Example – course change is 70 degrees, nominal bank angle of 25 degrees, no wind and TAS is 200kts

$$R = \frac{(102.889)^2}{9.81 \cdot tan(0.4363)} = 2{,}315m \qquad L2 = 5 \cdot (102.889) = 514m$$

$$L1 = R \cdot tan(0.5 \cdot 1.2217\ ) = 1{,}621m \qquad L = L1 + L2 \approx 2{,}135m = 1.2NM$$

Fly-over Waypoint



1. An initial roll-in at the flyover point; followed by
2. A straight 30 degree intercept course with next leg;
3. A roll-out at the new course; and
4. A 10-second delay to account for bank establishing time

Fly-By Waypoint Example

Table III-2-1-9. Minimum stabilization distance between fly-by waypoints (Non-SI units, 25° bank angle*)

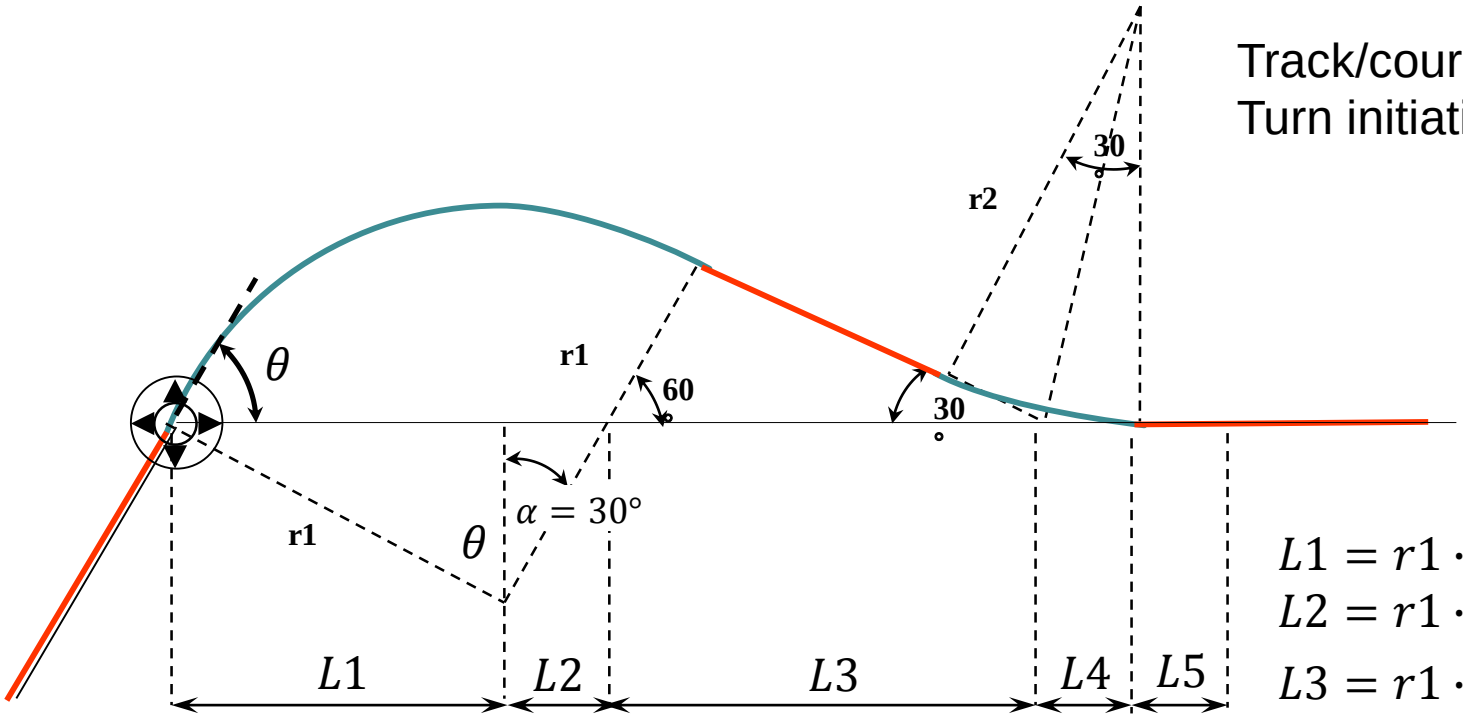
*Course change** (Degrees)*	*True airspeed (kt)*														
	< or = 130	*140*	*150*	*160*	*170*	*180*	*190*	*200*	*210*	*220*	*240*	*260*	*280*	*300*	*340*
50	0.5	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.2	1.3	1.5	1.7	2.2
55	0.5	0.6	0.6	0.7	0.7	0.8	0.9	0.9	1.0	1.1	1.3	1.5	1.7	1.9	2.4
60	0.5	0.6	0.7	0.7	0.8	0.8	0.9	1.0	1.1	1.2	1.4	1.6	1.8	2.0	2.6
65	0.5	0.7	0.7	0.8	0.8	0.9	1.0	1.1	1.2	1.3	1.5	1.7	1.9	2.2	2.8
70	0.6	0.7	0.8	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.6	1.8	2.1	2.4	3.0
75	0.6	0.8	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.5	1.7	2.0	2.3	2.6	3.2
80	0.7	0.8	0.9	0.9	1.0	1.1	1.2	1.3	1.4	1.6	1.8	2.1	2.4	2.8	3.5
85	0.7	0.9	0.9	1.0	1.1	1.2	1.3	1.4	1.6	1.7	2.0	2.3	2.6	3.0	3.8
90	0.7	0.9	1.0	1.1	1.1	1.3	1.4	1.5	1.7	1.8	2.1	2.5	2.8	3.2	4.1
95	0.8	1.0	1.1	1.1	1.2	1.4	1.5	1.6	1.8	2.0	2.3	2.7	3.1	3.5	4.4
100	0.8	1.1	1.2	1.2	1.3	1.5	1.6	1.8	1.9	2.1	2.5	2.9	3.3	3.8	4.8
105	0.9	1.2	1.2	1.3	1.4	1.6	1.7	1.9	2.1	2.3	2.7	3.1	3.6	4.1	5.2
110	1.0	1.3	1.3	1.4	1.5	1.7	1.9	2.1	2.3	2.5	2.9	3.4	3.9	4.4	5.6
115	1.1	1.4	1.5	1.6	1.7	1.8	2.0	2.2	2.5	2.7	3.2	3.7	4.2	4.8	6.1
120	1.2	1.5	1.6	1.7	1.8	2.0	2.2	2.4	2.7	2.9	3.5	4.0	4.6	5.3	6.7

* 25° or 3°/s

** Use the value of 50° for course changes lower than 50°.

See: ICAO Aircraft Operations – Volume II – Construction of Visual and Instrument Flight Procedures – Part III, Section 2, Chapter 1

Fly-over Waypoint



Track/course change: α
Turn initiation distance: L

$L1 = r1 \cdot sin\theta$

$L2 = r1 \cdot cos\theta \cdot tan\alpha$

$L3 = r1 \cdot \left[\frac{1}{sin\alpha} - \frac{2cos\theta}{\sin(90°-\alpha)}\right]$

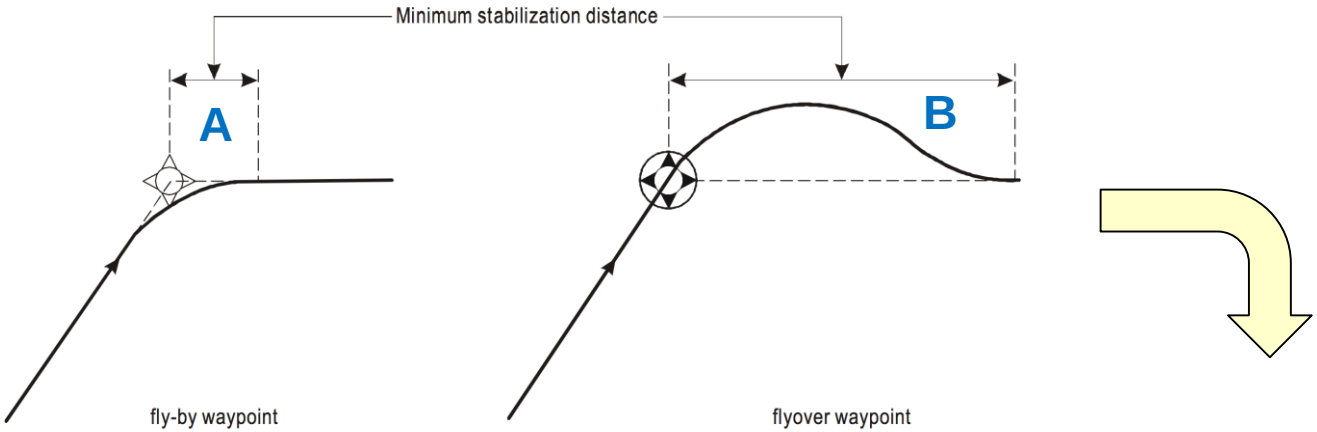
$L4 = r2 \cdot \tan\left(\frac{\alpha}{2}\right)$

$L5 = cV$

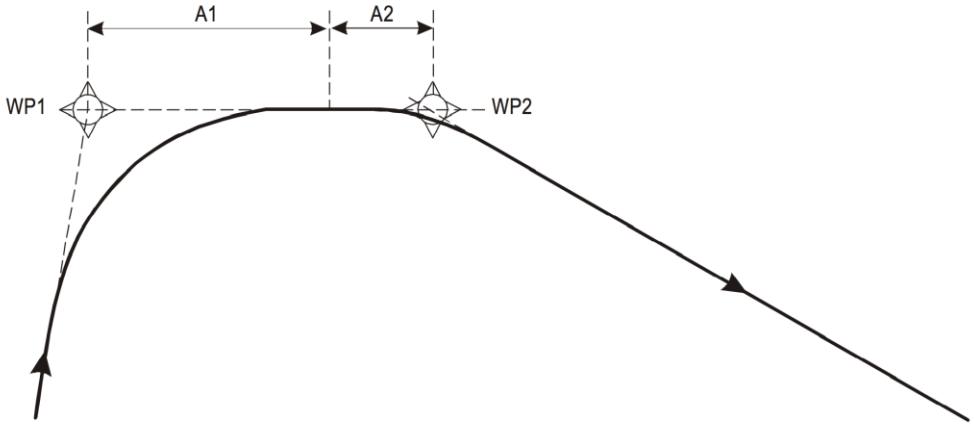
10 seconds bank establishment time

$r1$ = roll-in radius
$r2$ = roll-out radius

Waypoint Combinations

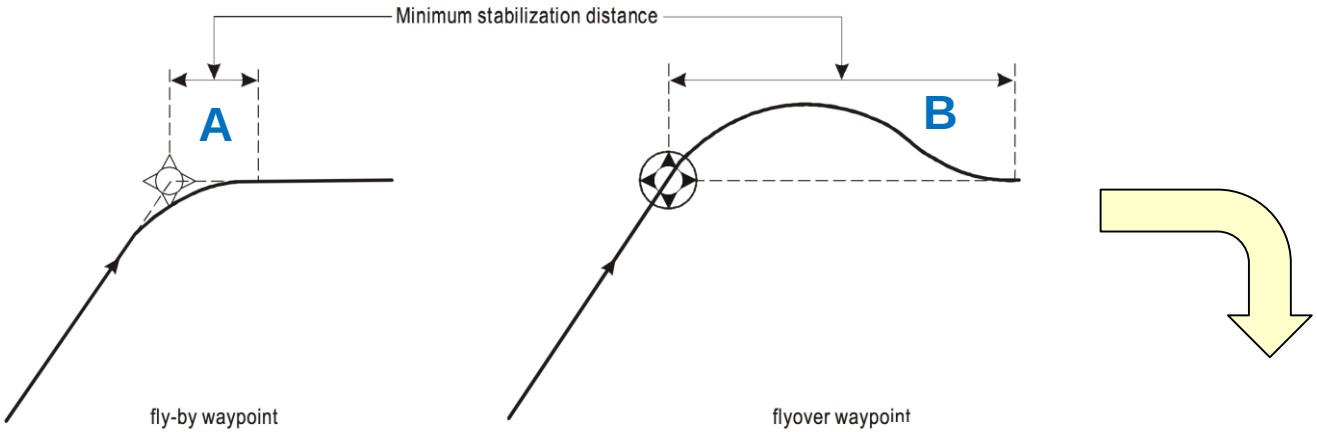


Two fly-by waypoints

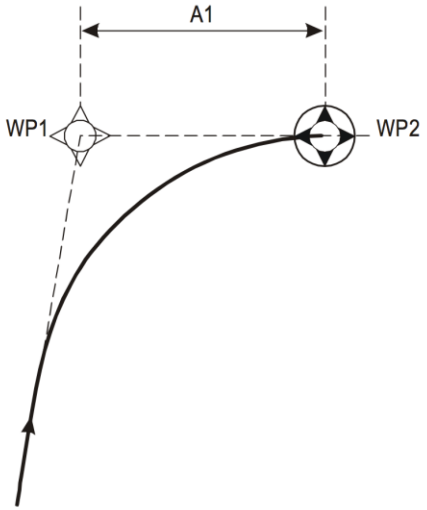


See: ICAO Aircraft Operations – Volume II – Construction of Visual and Instrument Flight Procedures – Part III, Section 2, Chapter 1

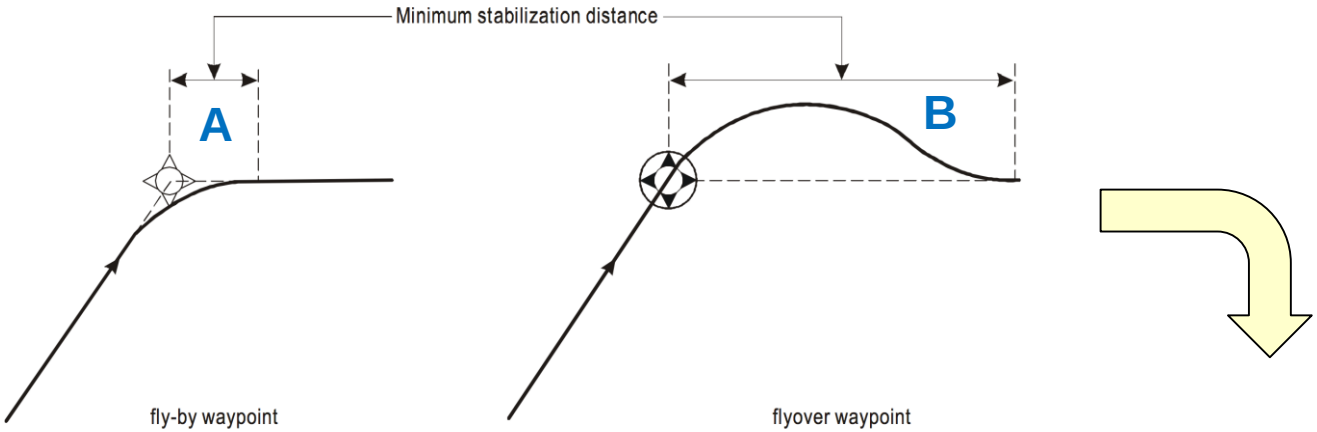
Waypoint Combination



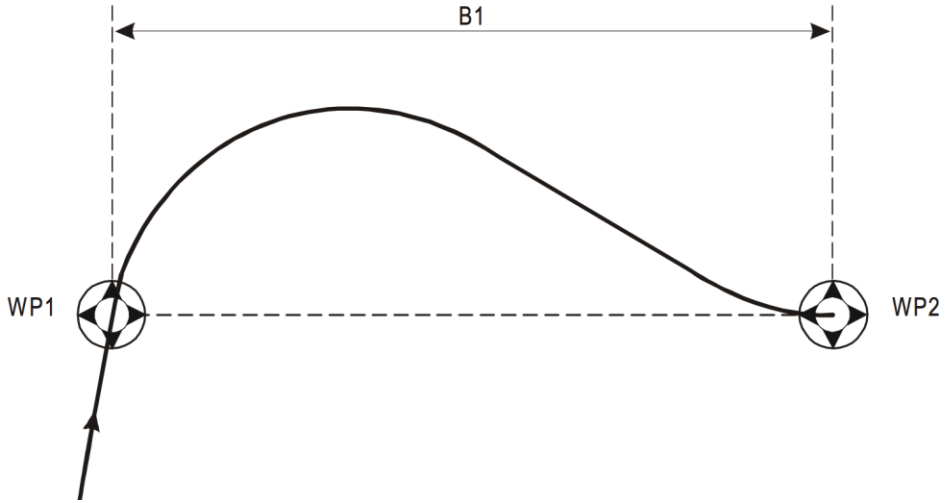
Fly-by then fly-over



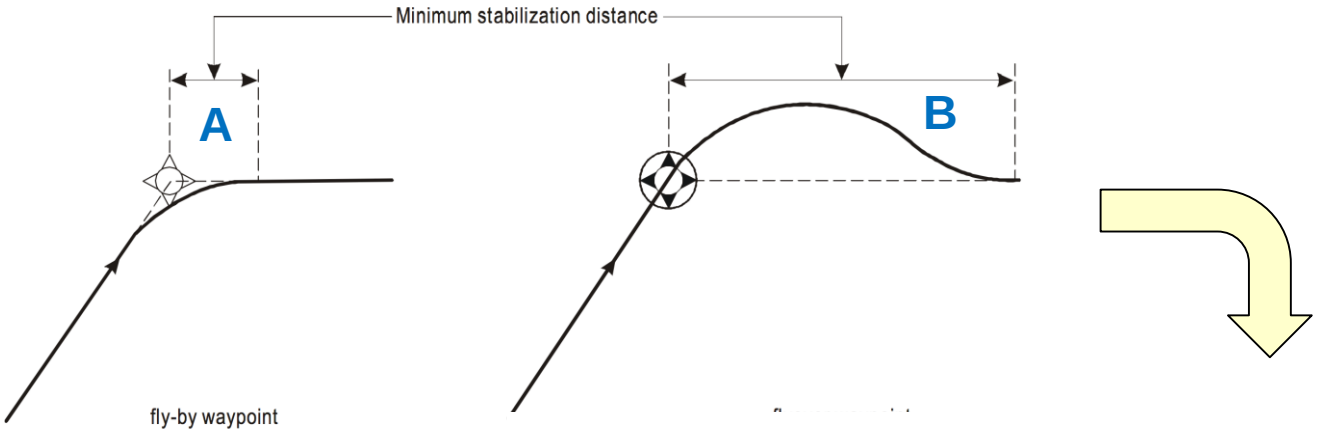
Waypoint Combination



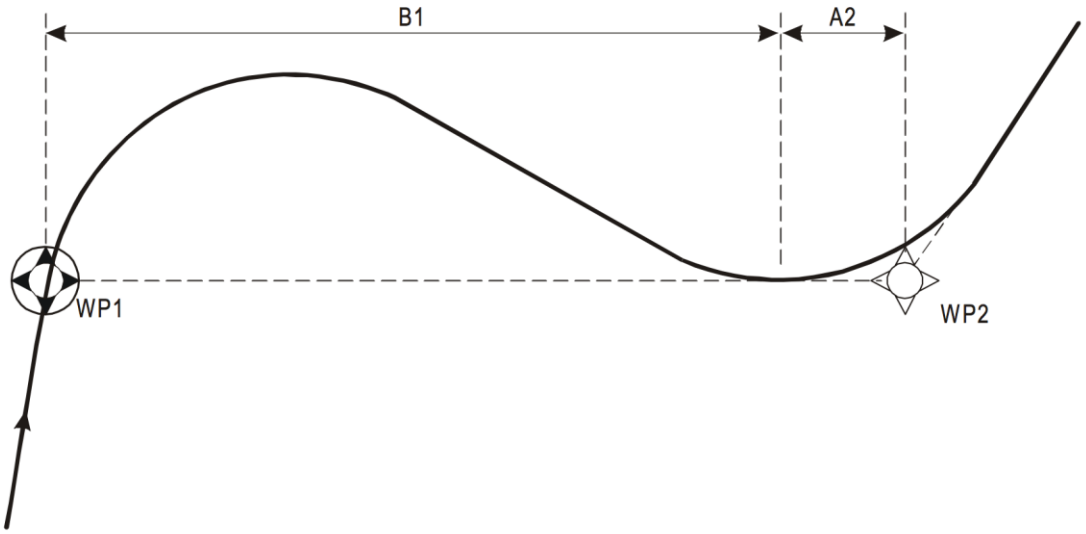
Two fly-over waypoints



Waypoint Combination



Fly-over then Fly-by

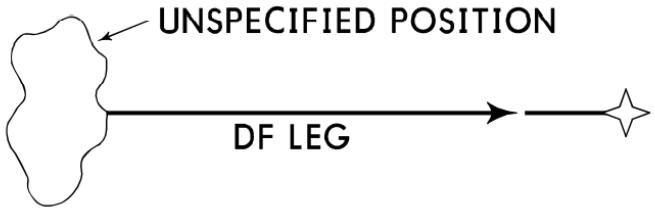
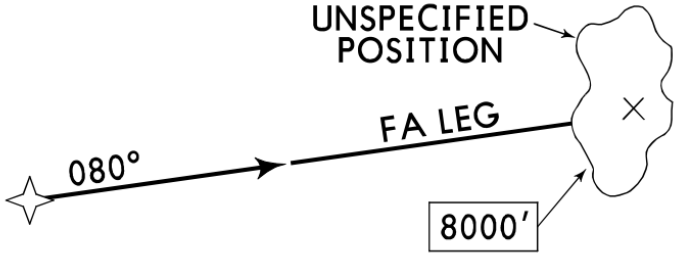


ARINC 424 Leg Types

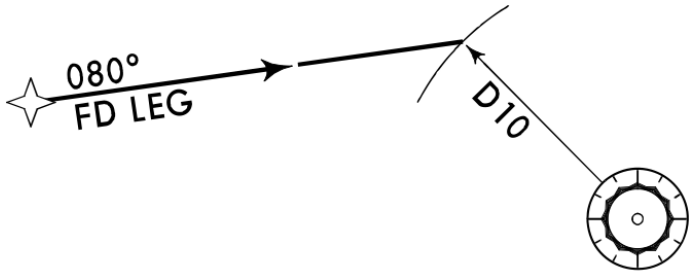
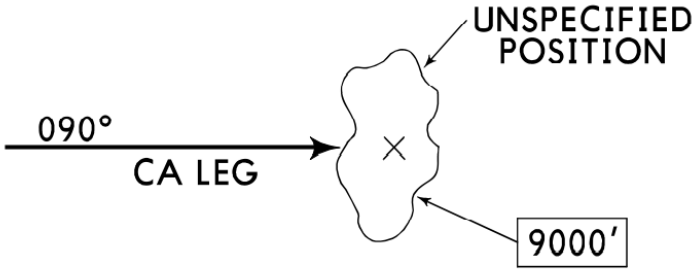
Leg Code	Example Path	Description
IF		Figure 1: Initial Fix or IF Leg. Defines a database fix as a point in space.
TF		Figure 2: Track to a Fix or TF Leg. Defines a great circle track over ground between two known databases fixes.
CF		Figure 3: Course to a Fix or CF Leg. Defines a specified course to a specific database fix.

intermediate and final approach segments should always be TF routes

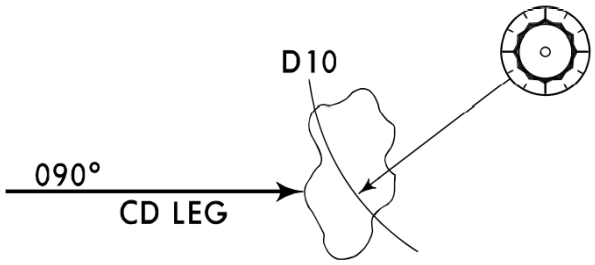
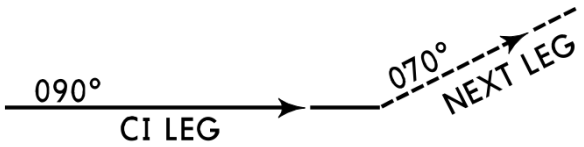
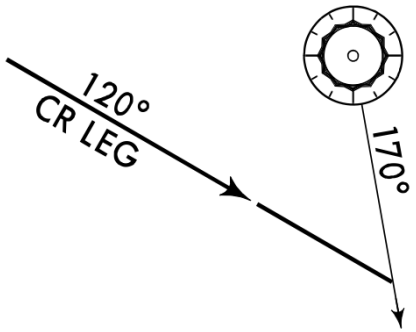
ARINC 424 Legs

DF		Figure 4: Direct to a Fix or DF Leg. Defines an unspecified track starting from an undefined position to a specific database fix. Note: See also Table 1.3, Leg Sequencing, for other uses of the DF Leg.
FA		Figure 5: Fix to an Altitude or FA Leg. Defines a specified track over ground from a database fix to a specified altitude at an unspecified position.
FC		Figure 6: Track from a Fix from a Distance or FC Leg. Defines a specified track over ground from a database fix for a specific distance.

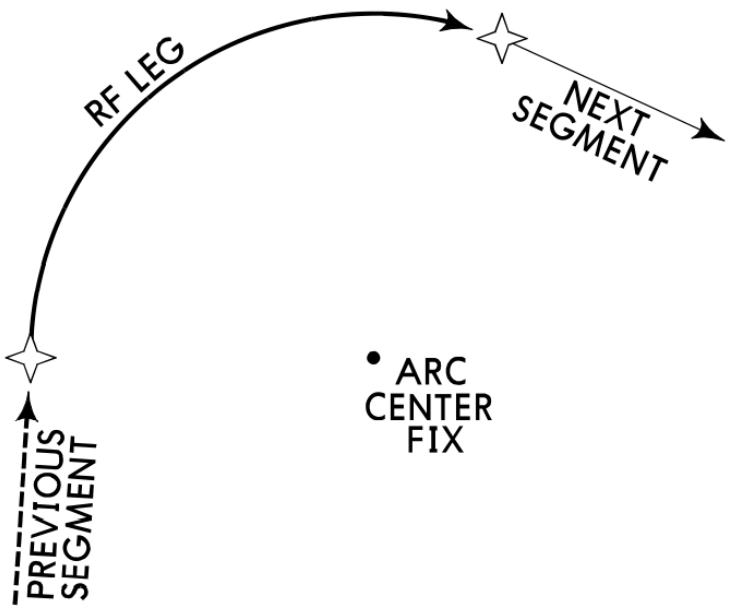
ARINC 424 Legs

FD		Figure 7: Track from a Fix to a DME Distance or FD Leg. Defines a specified track over ground from a database fix to a specific DME Distance which is from a specific database DME Navaid.
FM		Figure 8: From a Fix to a Manual termination or FM Leg. Defines a specified track over ground from a database fix until Manual termination of the leg.
CA		Figure 9: Course to an Altitude or CA Leg. Defines a specified course to a specific altitude at an unspecified position.

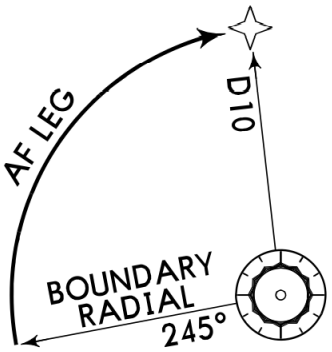
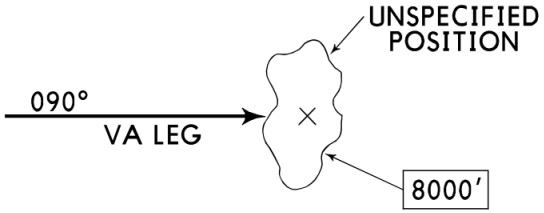
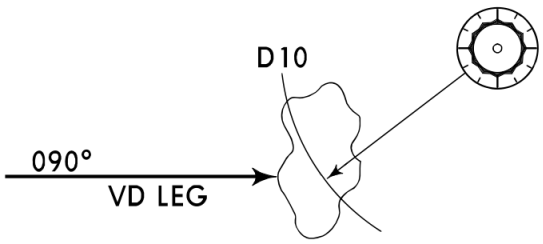
ARINC 424 Legs

CD		Figure 10: Course to a DME Distance or CD Leg. Defines a specified course to a specific DME Distance which is from a specific database DME Navaid.
CI		Figure 11: Course to an Intercept or CI Leg. Defines a specified course to intercept a subsequent leg.
CR		Figure 12: Course to a Radial termination or CR Leg. Defines a course to a specified Radial from a specific database VOR Navaid.

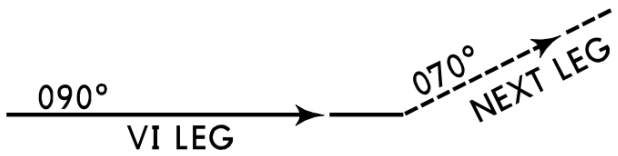
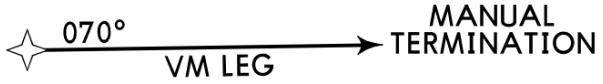
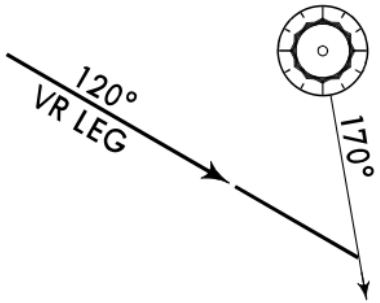
ARINC 424 Legs

RF		Figure 13: Constant Radius Arc or RF Leg. Defines a constant radius turn between two database fixes, lines tangent to the arc and a center fix. Note: While the arc initial point, arc ending point and arc centerpoint are all available as database fixes, implementation of this leg type may not require them to be available as fixes.

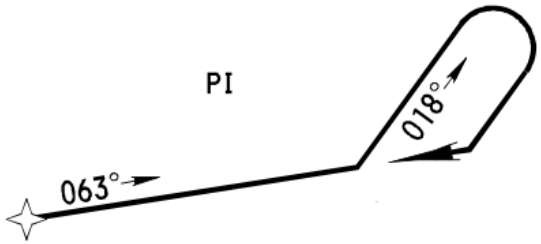
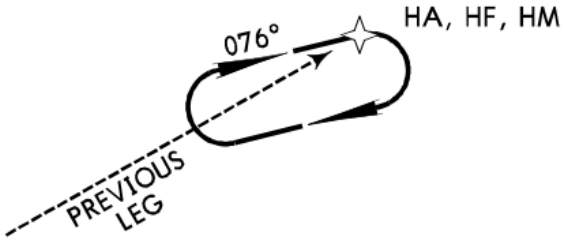
ARINC 424 Legs

AF		Figure 14: Arc to a Fix or AF Leg. Defines a track over ground at specified constant distance from a database DME Navaid.
VA		Figure 15: Heading to an Altitude termination or VA Leg. Defines a specified heading to a specific Altitude termination at an unspecified position.
VD		Figure 16: Heading to a DME Distance termination or VD Leg. Defines a specified heading terminating at a specified DME Distance from a specific database DME Navaid.

ARINC 424 Legs

VI		Figure 17: Heading to an Intercept or VI Leg. Defines a specified heading to intercept the subsequent leg at an unspecified position.
VM		Figure 18: Heading to a Manual termination or VM Leg. Defines a specified heading until a Manual termination.
VR		Figure 19: Heading to a Radial termination or VR Leg. Defines a specified heading to a specified radial from a specific database VOR Navaid.

ARINC 424 Legs

PI		Figure 20: 045/180 Procedure Turn or PI Leg. Defines a course reversal starting at a specific database fix, includes Outbound Leg followed by a left or right turn and 180 degree course reversal to intercept the next leg. A Maximum excursion Time or Distance is included as a data field.
HA, HF, HM		Figure 21: Holding in lieu of Procedure Turn (HF) for Approach Procedures and Mandatory Holds (HA, HM) in SID/STAR and Missed Approach coding. The HA, HF, and HM Leg Types define a holding pattern in lieu of procedure turn course reversal or a terminal procedure referenced mandatory holding pattern at a specified database fix. Leg time or distance is included as a data field. The three codes indicate different path termination types: HA = Altitude Termination HF = Single circuit terminating at the fix. HM = Manual Termination.

Data Required
per ICAO

Table III-2-5-App-3. Path terminators (Required data)

Path terminator	*Waypoint identifier*	*Flyover*	*Turn direction*	*Recommended Navaid*	*Distance from Navaid*	*Bearing from Navaid*	*Magnetic course*	*Path length*	*Altitude restriction 1*	*Altitude restriction 2*	*Speed limit*	*Vertical angle*	*Arc centre*
CA			O				✓		6		O		
CF	✓	1	O	✓	✓	✓	✓		O	O	O	O	
DF	✓	1	O	O	O	O			O	O	O		
FA	✓		O	✓	✓	✓	✓		6		O		
FM	✓		O	✓	✓	✓	✓		O		O		
HA	✓		O	O	O	O	✓	✓	6		O		
HF	✓		O	O	O	O	✓	✓	O		O		
HM	✓		O	O	O	O	✓	✓	O		O		
IF	✓			O	O	O			O	O	O		
RF	✓	O	✓	O		2	3	5	O	O	O	O	✓
TF	✓	O	O	O	O	O	O	O	O	O	O	O	
VA			O				4		6		O		
VI		O	O	O			4		O	O	O		
VM	O		O				4		O		O		

✓ — Required
O — Optional
1 — Required for CF/DF and DF/DF combinations only.
2 — Inbound tangential track
3 — Outbound tangential track
4 — Heading not course
5 — Along track distance
6 — Altitude at or above

Data Required
per ARINC

Leg Data Fields Table 3

PT	W/P ID	OVR FLY	HLD	TD	TDV	RMD NAV	THETA	RHO	OBD MAG CRS	TM/ DST	ALT ONE	ALT TWO	SPD LMT (1)	VRT ANG	ARC CTR	COMMENTS
AF	X	O	O	X		X	X	X	R		O	O	O			OB MAG CRS=BNDY RDL, THETA=FIX RDL
CA				O	O				C		+		O			ALT TERM WILL BE "AT OR ABOVE"
CD				O	O	X			C	D	O	O	O			
CF	X	B	O	O	O	X	X	X	C	F	O	O	O	O		OB MAG CRS IS CRS TO SPECIFIED FIX
CI		O		O	O	O			C		O	O	O			
CR		O		O	O	X	X		C		O	O	O			
DF	X	B	O	O		O	O	O			O	O	O			
FA	X			O	O	X	X	X	C		+		O			ALT TERM WILL BE "AT OR ABOVE"
FC	X	B		O	O	X	X	X	C	P	O	O	O			
FD	X	O		O	O	X	X	X	C	D	O	O	O			
FM	X			O	O	X	X	X	C		O		O			
HA	X	O		X		O	O	O	C	X	+		O			ALT TERM WILL BE "AT OR ABOVE"
HF	X	O		X		O	O	O	C	X	O		O			
HM	X	O		X		O	O	O	C	X	O		O			
IF	X		O			O	O	O			O	O	O			
PI	X			X		X	X	X	C	P	X		O			DIST IS EXCURSION DIST FROM FIX
RF	X		O	X		O	O		O	A	O	O	O	O	X	
TF	X	B	O	O	O	O	O	O	O	O	O	O	O	O		
VA				O	O				H		+		O			ALT TERM WILL BE "AT OR ABOVE"
VD				O	O	X			H	D	O	O	O			
VI		O		O	O	O			H		O	O	O			
VM	O			O	O				H		O		O			FOR W/P ID SEE STAR CODING RULES
VR		O		O	O	X	X		H		O	O	O			

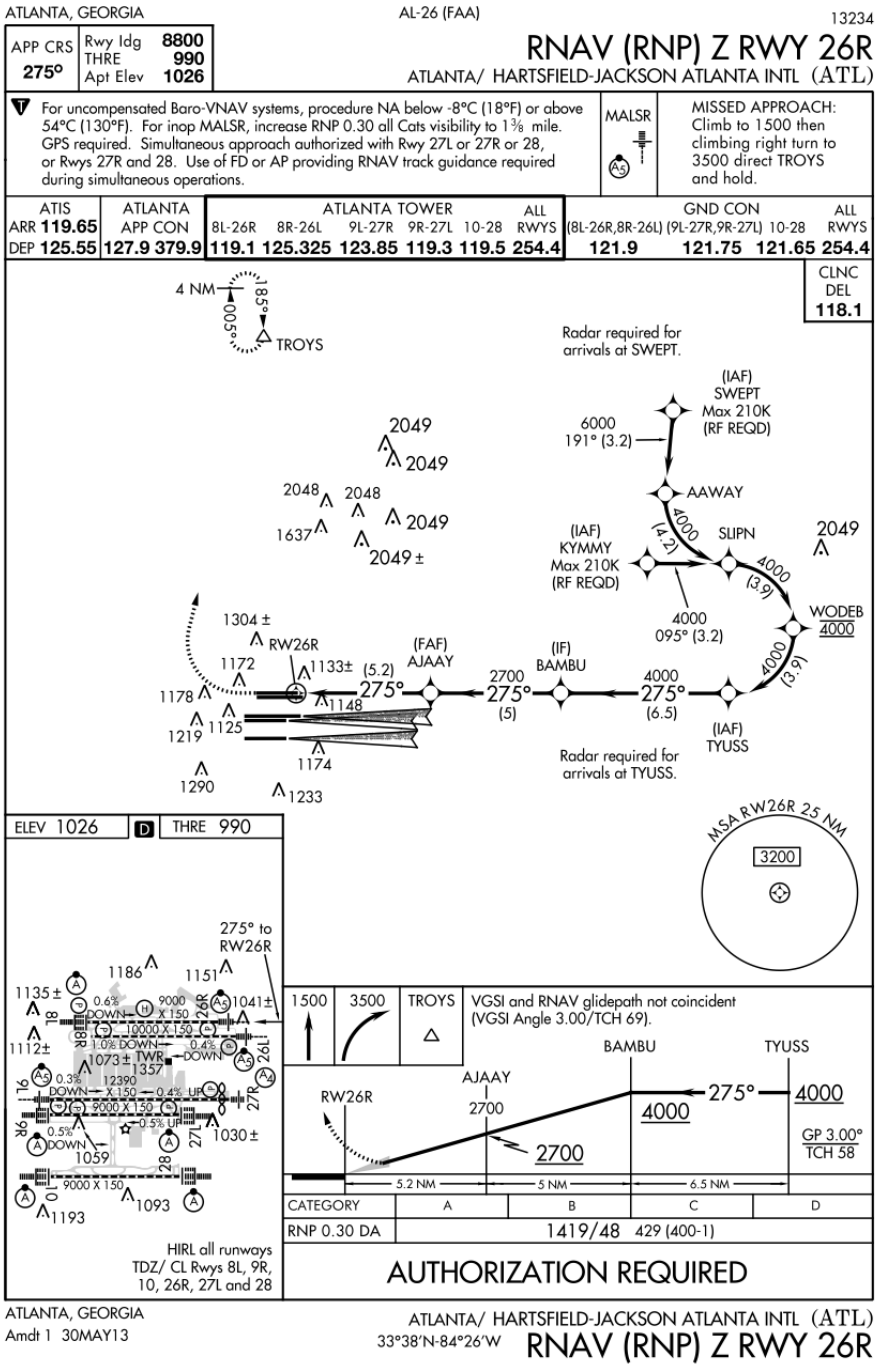
LEGEND:

X = REQUIRED FIELD
A = ALONG TRACK DISTANCE
O = OPTIONAL FIELD
P = PATH LENGTH
R = BOUNDARY RADIAL
C = COURSE
H = HEADING
D = DME DISTANCE
+ = "AT OR ABOVE" ONLY
SHADED = NOT APPLICABLE FIELD
B = "REQUIRED" FOR CF/DF, DF/DF, TF/DF OR FC/DF COMBINATIONS, OTHERWISE "OPTIONAL"

Path Terminators

Table III-2-5-App-1. Initial and final path terminators

RNAV procedure	*Initial leg*	*Final leg*
SID	CA, CF, VA, VI	CF, DF, FM, HA, RF, TF, VM
STAR	IF	CF, DF, FM, HM, RF, TF, VM
Approach	IF	CF, TF, RF
Missed approach	CA, CF, DF, FA, HA, HM, RF, VI, VM	CF, DF, FM, HM, RF, TF, VM

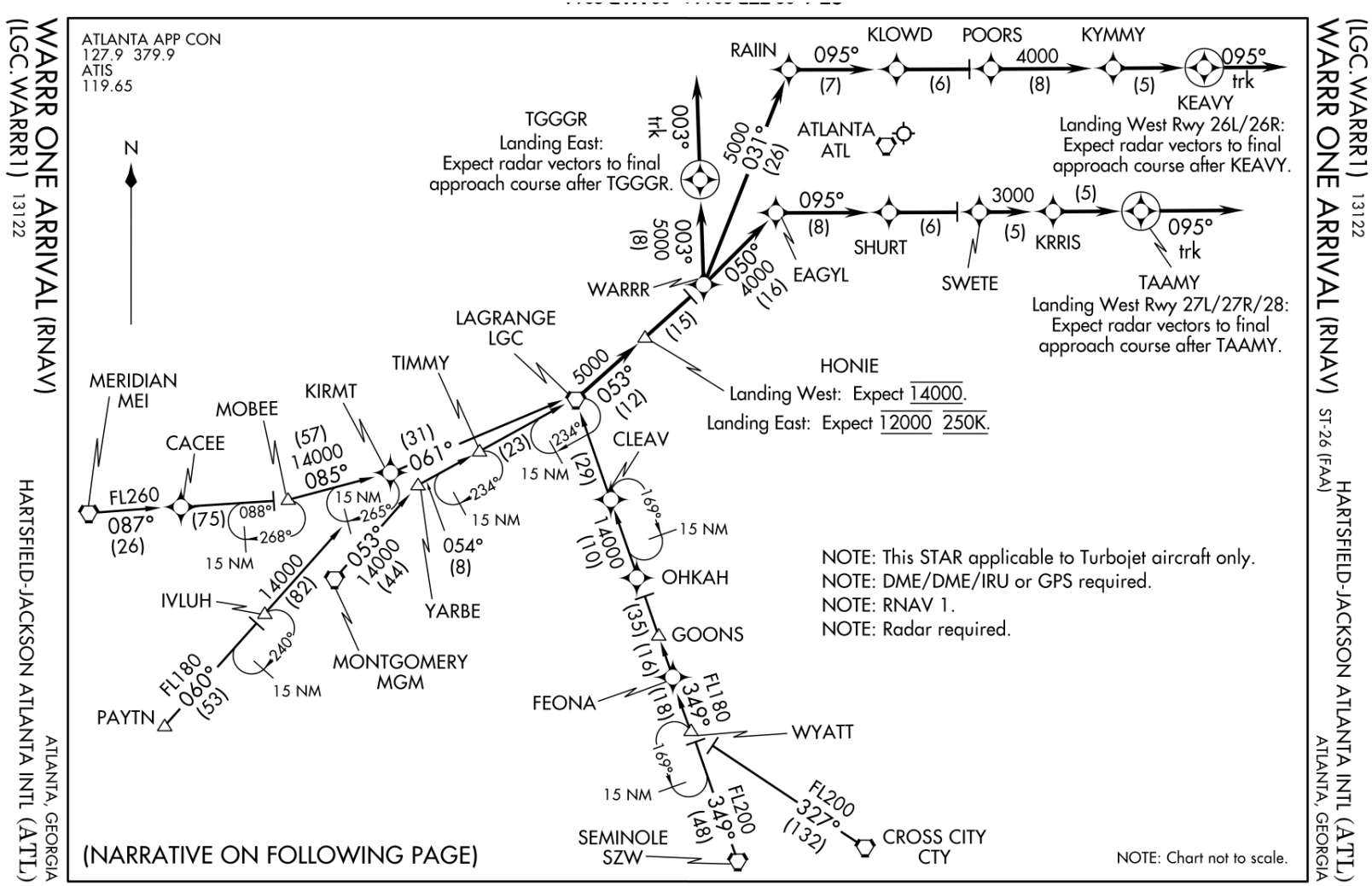


Approach

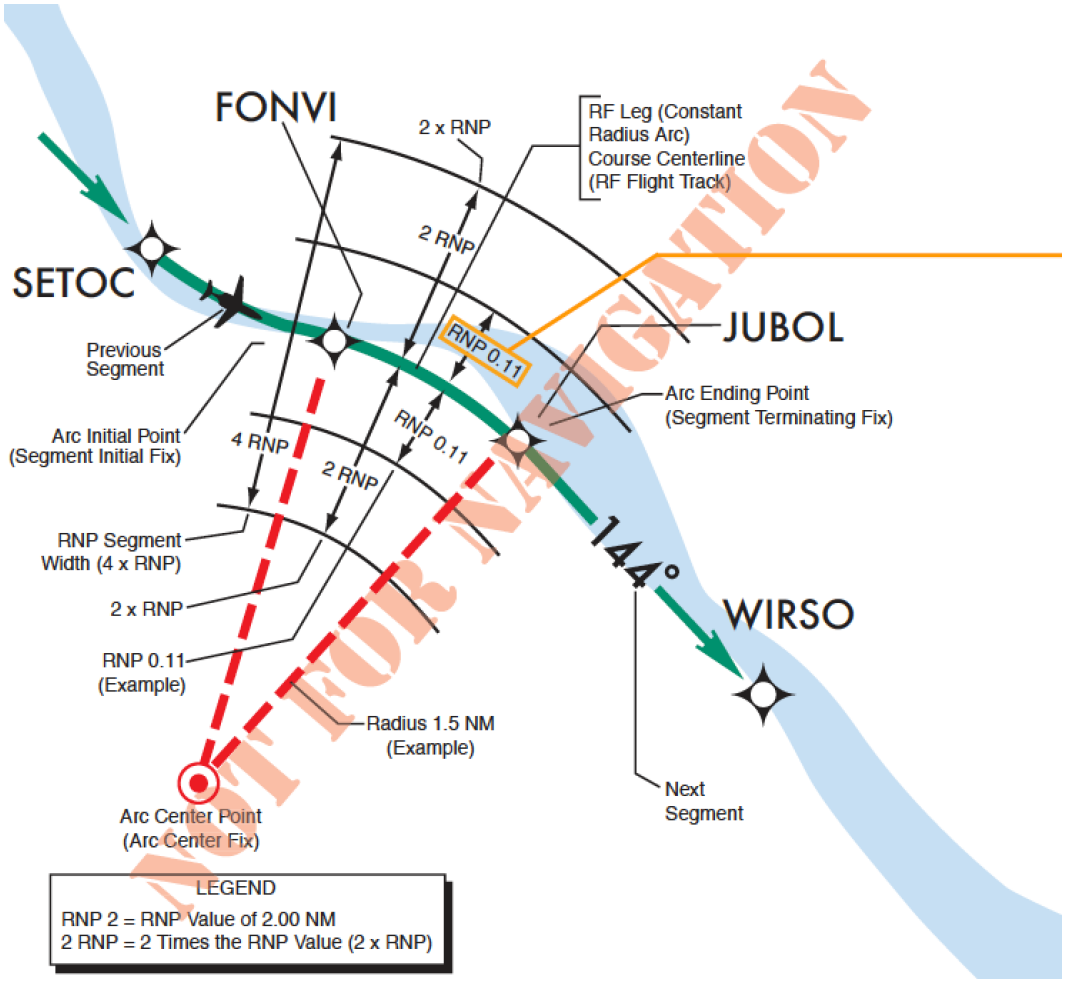
Table III-2-5-App-1. Initial and final path terminators

RNAV procedure	*Initial leg*	*Final leg*
SID	CA, CF, VA, VI	CF, DF, FM, HA, RF, TF, VM
STAR	IF	CF, DF, FM, HM, RF, TF, VM
Approach	IF	CF, TF, RF
Missed approach	CA, CF, DF, FA, HA, HM, RF, VI, VM	CF, DF, FM, HM, RF, TF, VM

STAR into Atlanta



RF Leg plus RNP Example



Waypoint to Waypoint: Geodesics

- Geodesics:
  - Shortest distance between two points in curved spaces;
- Examples:
  - Plane: straight line
  - Sphere: great circle
  - Ellipsoid like WGS 84: ….

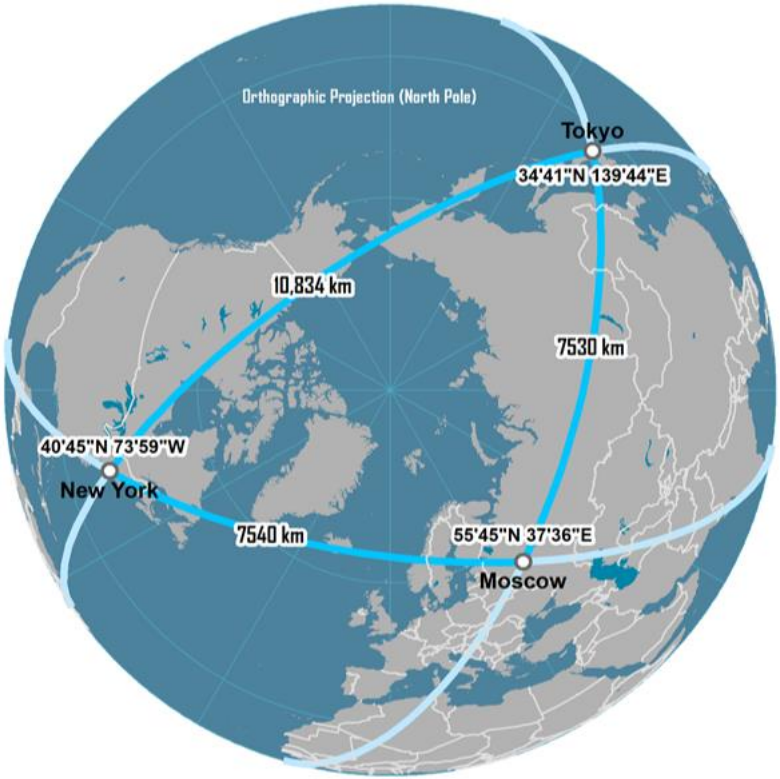
Used in GPS:

major (equatorial) radius a = 6378137 m at the equator
flattening f = 1/298.257223563

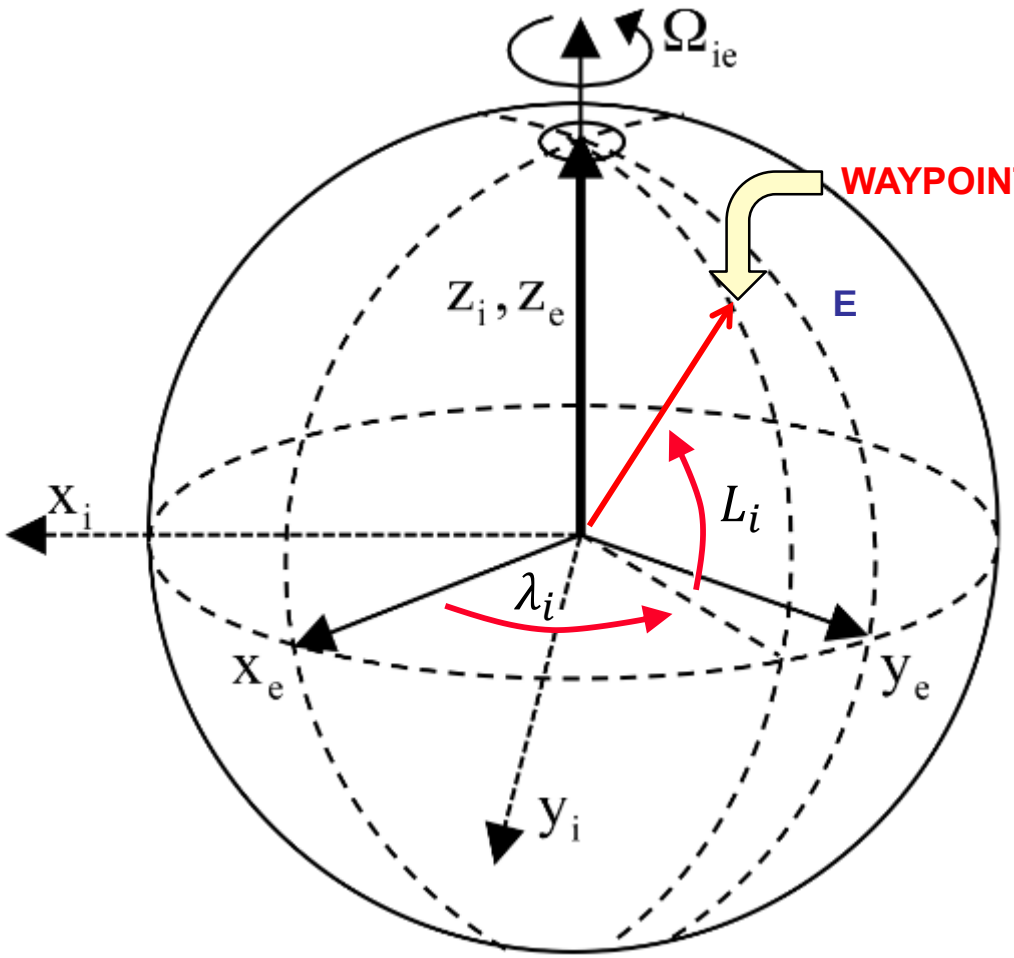
WGS = World Geodetic System

From Waypoint to Waypoint

- Great Circle Routes:
  - Assume Earth is a sphere



Great Circle Routes

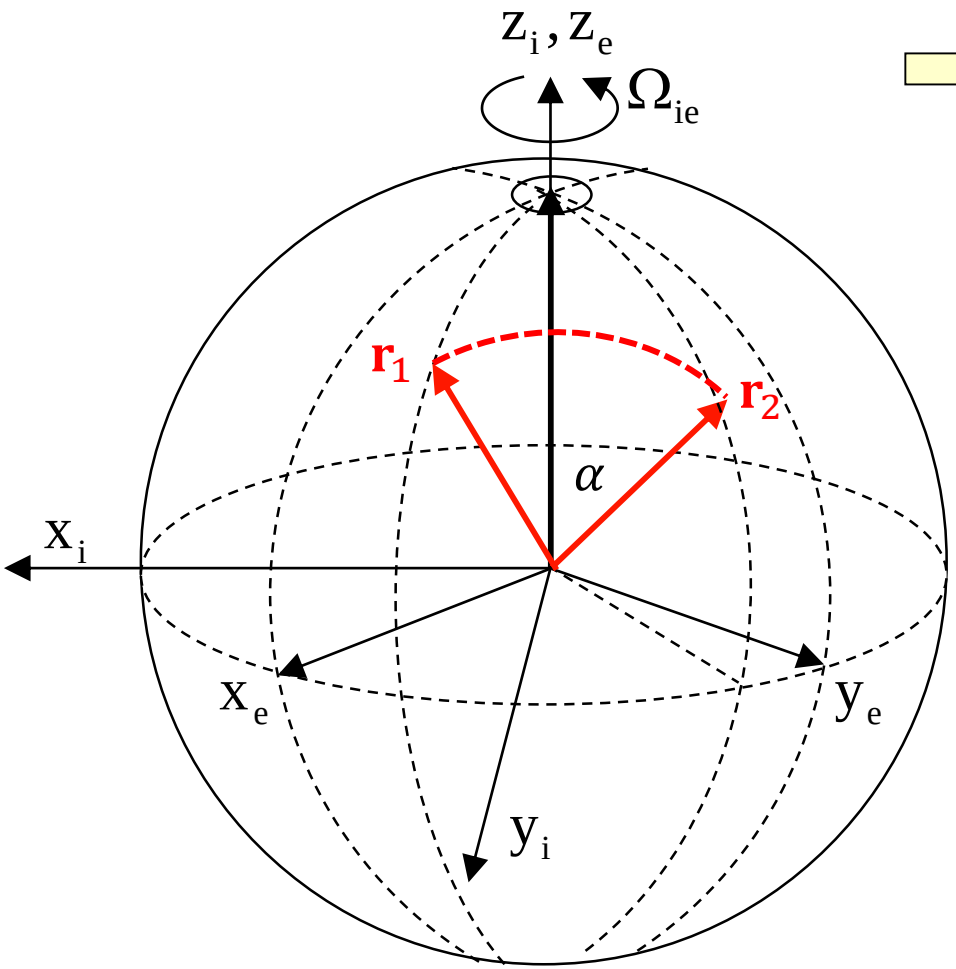


$$\mathbf{r}_i = R_e \begin{bmatrix} cos\lambda_i cosL_i \\ sin\lambda_i cosL_i \\ sinL_i \end{bmatrix} = R_e \mathbf{e}_i$$

Radius of a sphere
(approximate Earth by a sphere)

L_i = **waypoint latitude**
λ_i = **waypoint longitude**

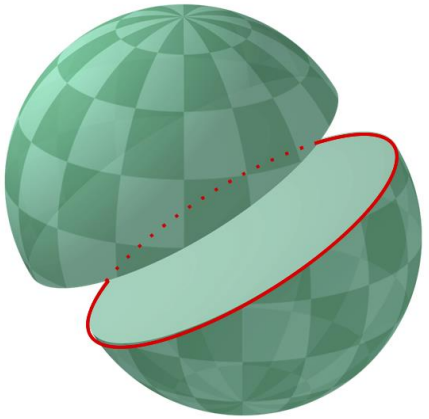
Great Circle Routes



$$\cos(\alpha) = \mathbf{e}_1 \cdot \mathbf{e}_2$$

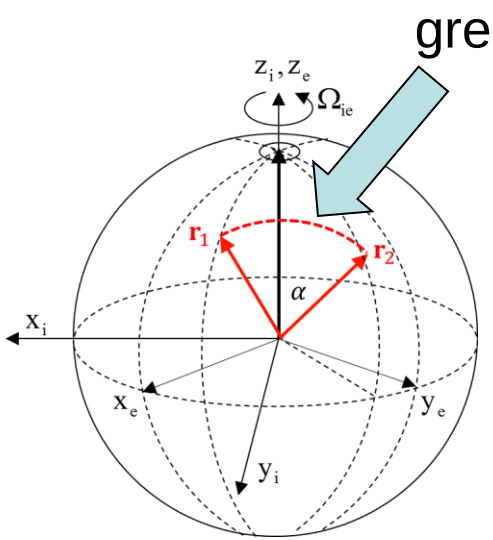
$$\sin(\alpha) = |\mathbf{e}_1 \times \mathbf{e}_2|$$

$$\tan(\alpha) = \left(\frac{|\mathbf{e}_1 \times \mathbf{e}_2|}{\mathbf{e}_1 \cdot \mathbf{e}_2}\right)$$



http://en.wikipedia.org/wiki/File:Great_circle_hemispheres.png

Great Circle Distance



$$\cos(\alpha) = \mathbf{e}_1 \cdot \mathbf{e}_2 \Rightarrow$$

$$\cos(\alpha) = \begin{bmatrix} cos\lambda_1 cosL_1 \\ sin\lambda_1 cosL_1 \\ sinL_1 \end{bmatrix} \cdot \begin{bmatrix} cos\lambda_2 cosL_2 \\ sin\lambda_2 cosL_2 \\ sinL_2 \end{bmatrix} \Rightarrow$$

$$\cos(\alpha) = cos\lambda_1 cosL_1 cos\lambda_2 cosL_2 + sin\lambda_1 cosL_1 sin\lambda_2 cosL_2 + sinL_1 sinL_2 \Rightarrow$$

$$\cos(\alpha) = cosL_1 cosL_2 [sin\lambda_1 sin\lambda_2 + cos\lambda_1 cos\lambda_2] + sinL_1 sinL_2 \Rightarrow$$

$$\cos(\alpha) = cosL_1 cosL_2 cos(\lambda_1 - \lambda_2) + sinL_1 sinL_2 \Rightarrow$$

$$\alpha = cos^{-1}\{cosL_1 cosL_2 cos(\lambda_1 - \lambda_2) + sinL_1 sinL_2\}$$

Spherical law of cosines

$$d = R_e \alpha = R_e cos^{-1}\{cosL_1 cosL_2 cos(\lambda_1 - \lambda_2) + sinL_1 sinL_2\}$$

Great Circle Distance

Let's rewrite this equation using identities:

$$sin^2\left(\frac{\gamma}{2}\right) = \frac{1}{2} - \frac{1}{2}cos\gamma \Rightarrow$$

$$cos\gamma = 1 - 2sin^2\left(\frac{\gamma}{2}\right)$$

and $cos\alpha cos\beta + \sin\alpha sin\beta = cos(\alpha - \beta)$

$$cosL_1 cosL_2 cos(\lambda_1 - \lambda_2) + sinL_1 sinL_2 \Rightarrow$$

$$cosL_1 cosL_2 - 2cosL_1 cosL_2 \sin^2(\lambda_1 - \lambda_2) + sinL_1 sinL_2 \Rightarrow$$

$$cosL_1 cosL_2 + sinL_1 sinL_2 - 2cosL_1 cosL_2 \sin^2(\lambda_1 - \lambda_2) \Rightarrow$$

$$cos(L_1 - L_2) - 2cosL_1 cosL_2 \sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right) \Rightarrow$$

$$1 - 2\sin^2\left(\frac{L_1 - L_2}{2}\right) - 2cosL_1 cosL_2 \sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right)$$

Great Circle Distance

$$\cos(\alpha) = 1 - 2\sin^2\left(\frac{L_1 - L_2}{2}\right) - 2cosL_1cosL_2\sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right) \Rightarrow$$

$$\cos(\alpha) = 1 - 2\left\{\sin^2\left(\frac{L_1 - L_2}{2}\right) + cosL_1cosL_2\sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right)\right\} \Rightarrow$$

$$1 - 2sin^2\left(\frac{\alpha}{2}\right) = 1 - 2\left\{\sin^2\left(\frac{L_1 - L_2}{2}\right) + cosL_1cosL_2\sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right)\right\} \Rightarrow$$

$$sin^2\left(\frac{\alpha}{2}\right) = \sin^2\left(\frac{L_1 - L_2}{2}\right) + cosL_1cosL_2\sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right) \Rightarrow$$

Haversine formula:

$$d = R_e\alpha = R_e 2arcsin\left(\sqrt{\sin^2\left(\frac{L_1 - L_2}{2}\right) + cosL_1cosL_2\sin^2\left(\frac{\lambda_1 - \lambda_2}{2}\right)}\right)$$

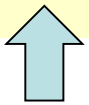
Is numerically better conditioned for small distances!

Great Circle Distance

- Even better is the Vincenty formula:

$$d = R_e \alpha$$

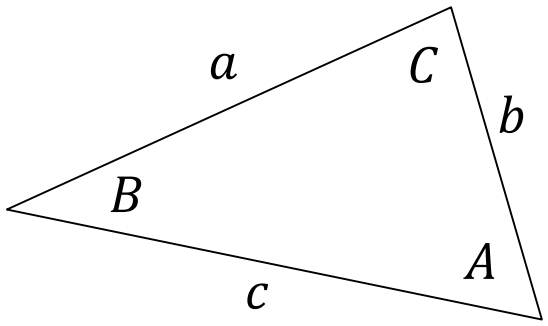
$$= R_e arctan\left(\frac{\sqrt{(\cos L_2 sin[\lambda_1 - \lambda_2])^2 + (cosL_1 sinL_2 - sinL_1 cosL_2 cos[\lambda_1 - \lambda_2])^2}}{cosL_1 cosL_2 cos(\lambda_1 - \lambda_2) + sinL_1 sinL_2}\right)$$

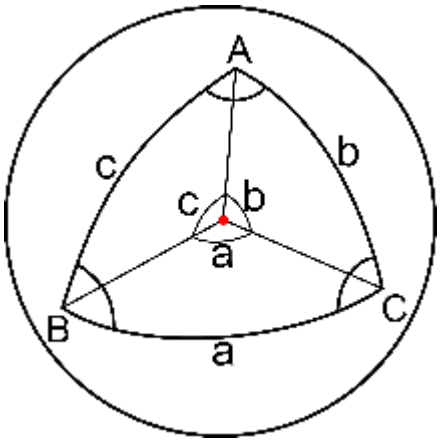


Use ‘atan2’ when implementing

- Accurate for most distances.

Regular Trigonometry





Law of Sines (sine rules):

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$

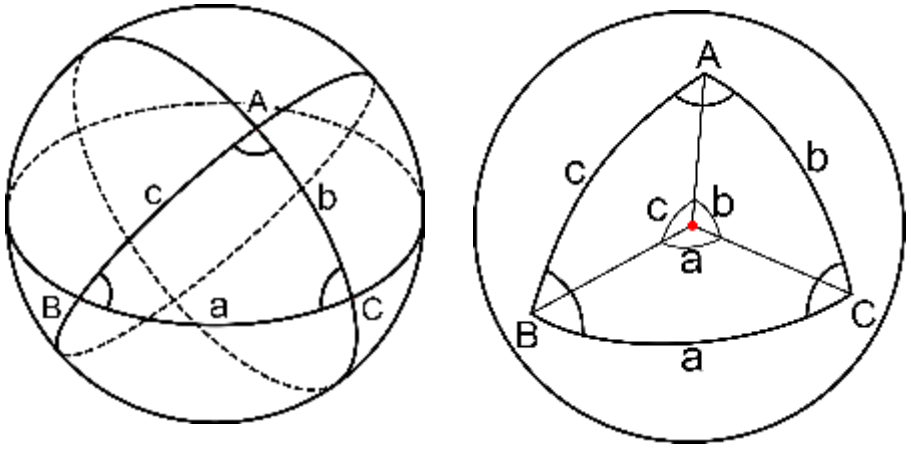
Law of Cosines (cosine rules):

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$
$$b^2 = a^2 + c^2 - 2ac\cos(B)$$
$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

A **spherical triangle** is one whose sides are all great circular arcs

Spherical Trigonometry

Unit spheres:



Sine rules:

$$\frac{sinA}{sina} = \frac{sinB}{sinb} = \frac{sinC}{sinc}$$

Cosine rules:

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$
$$\cos(b) = \cos(c)\cos(a) + \sin(c)\sin(a)\cos(B)$$
$$\cos(c) = \cos(a)\cos(b) + \sin(a)\sin(b)\cos(C)$$

Or:

$$\cos(A) = \cos(B)\cos(C) + \sin(B)\sin(C)\cos(a)$$
$$\cos(B) = \cos(C)\cos(A) + \sin(C)\sin(A)\cos(b)$$
$$\cos(C) = \cos(A)\cos(B) + \sin(A)\sin(B)\cos(c)$$

Images taken from Wikipedia

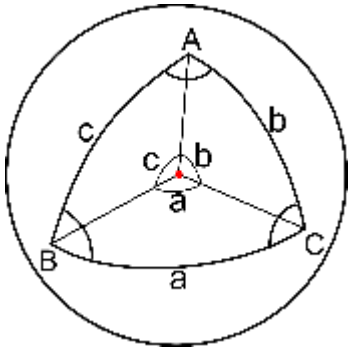
Spherical Trigonometry

Using law of cosines and sines:

$$\tan(A) = \sin(B)\sin(a)\,/[\sin(c)\cos(a) - \cos(B)\cos(c)\sin(a)\,]$$
$$\tan(B) = \sin(C)\sin(b)\,/[\sin(a)\cos(b) - \cos(C)\cos(a)\sin(b)\,]$$
$$\tan(C) = \sin(A)\sin(c)\,/[\sin(b)\cos(c) - \cos(A)\cos(b)\sin(c)\,]$$

And:

$$\tan(a) = \sin(b)\sin(A)\,/[\sin(C)\cos(A) - \cos(b)\cos(C)\sin(A)\,]$$
$$\tan(b) = \sin(c)\sin(B)\,/[\sin(A)\cos(B) - \cos(c)\cos(A)\sin(B)\,]$$
$$\tan(c) = \sin(a)\sin(C)\,/[\sin(B)\cos(C) - \cos(a)\cos(B)\sin(C)\,]$$

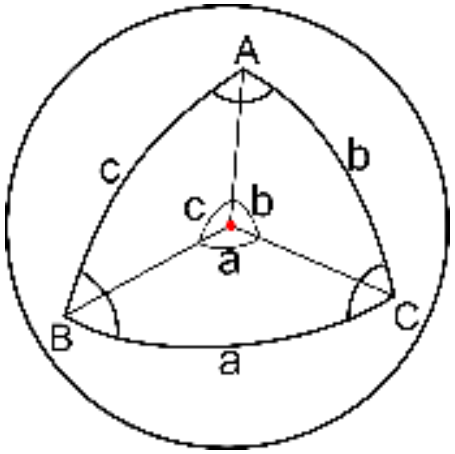


Spherical Trigonometry Solution

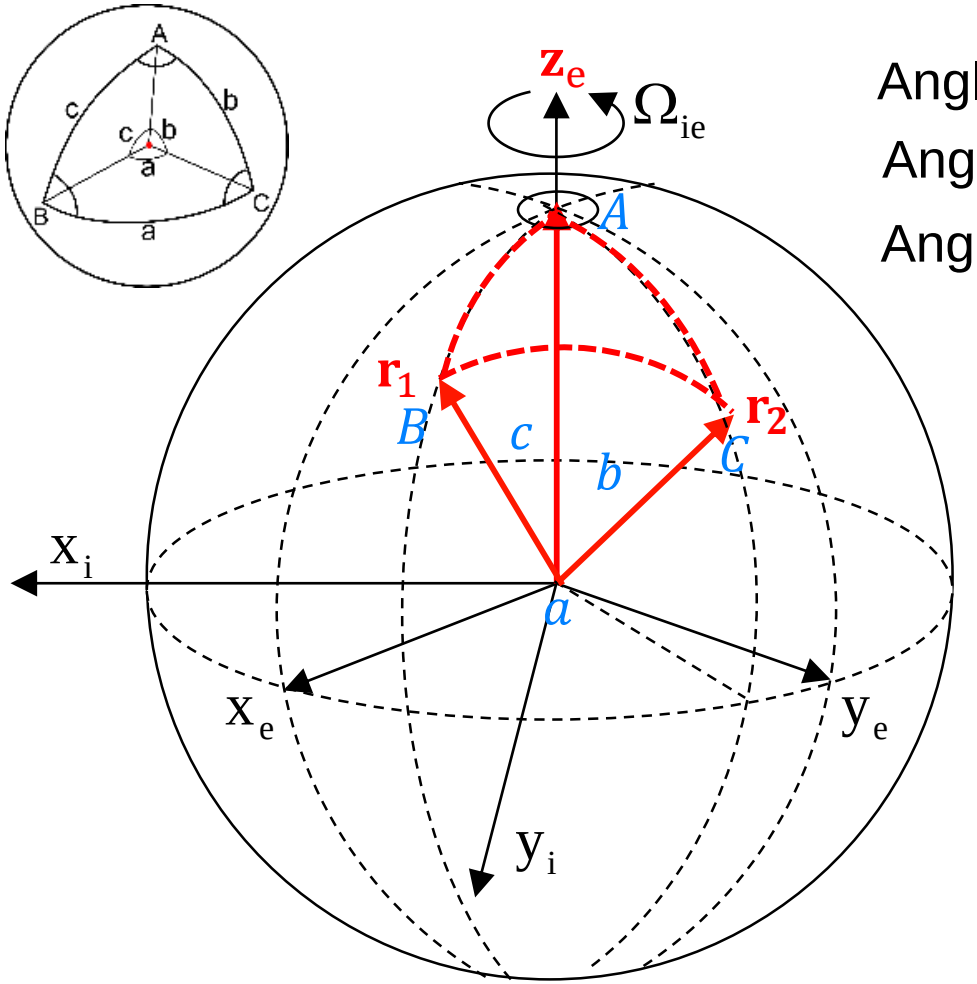
Given the "sides" $\{a, b, c\}$, one can find the "angles":

$$A = \text{acos}\{(\cos(a) - \cos(b)\cos(c))/(\sin(b)\sin(c))\}$$
$$B = \text{acos}\{(\cos(b) - \cos(c)\cos(a))/(\sin(c)\sin(a))\}$$
$$C = \text{acos}\{(\cos(c) - \cos(a)\cos(b))/(\sin(b)\sin(b))\}$$

And various others as shown in:
Aviation Formulary V1.46 by Williams



Spherical Trigonometry Applied



Angle between $\mathbf{r}_1$ and $\mathbf{r}_2$ is $\alpha \ (= a)$

Angle between $\mathbf{r}_1$ and $\mathbf{z}_e$ is $90° - L_1 \ (= c)$

Angle between $\mathbf{r}_2$ and $\mathbf{z}_e$ is $90° - L_2 (= b)$

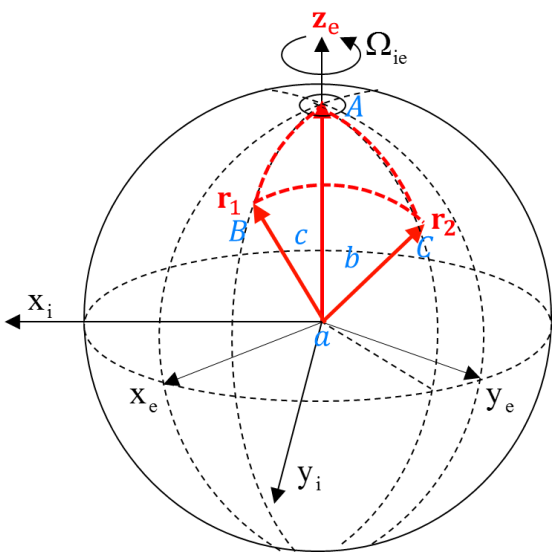
Remember also:

$$\cos(90° - x) = \sin(x)$$
$$\sin(90° - x) = \cos(x)$$

Then:

$$\cos(c) = \cos(90° - L_1) = \sin(L_1)$$
$$\sin(c) = \sin(90° - L_1) = \cos(L_1)$$
$$\cos(b) = \cos(90° - L_2) = \sin(L_2)$$
$$\sin(b) = \sin(90° - L_2) = \cos(L_2)$$

Great Circle Route Initial Course



Course between two waypoints at first waypoint:

$$if\ \cos(\lambda_1 - \lambda_2) < 0$$

$$tc_1 = \operatorname{acos}\left(\frac{\sin(L_2) - \sin(L_1)\cos(\alpha)}{\cos(L_1)\sin(\alpha)}\right)$$

$$else$$

$$tc_1 = 2\pi - \operatorname{acos}\left(\frac{\sin(L_2) - \sin(L_1)\cos(\alpha)}{\cos(L_1)\sin(\alpha)}\right)$$

$$endif$$

When first waypoint is at one of the poles:

$$if\ \cos(L_1) < eps\ (small\ number)$$

$$if\ L_1 > 0$$

$$tc_1 = \pi$$

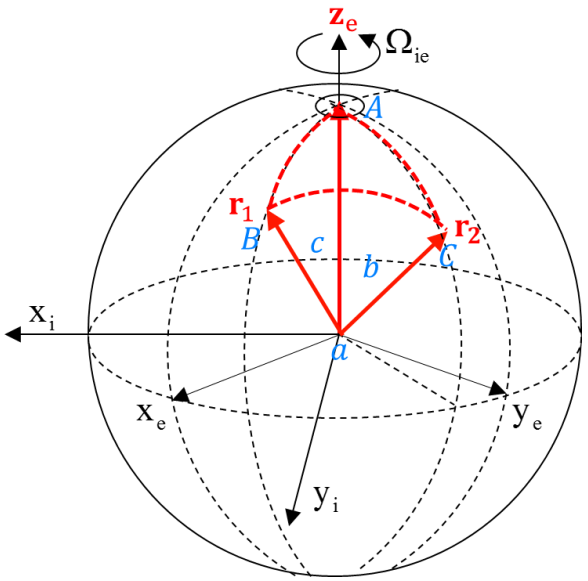
$$else$$

$$tc_1 = 2\pi$$

$$endif$$

$$endif$$

Great Circle Route Initial Course



Course between two waypoints at first waypoint:

$$if\ \cos(\lambda_1 - \lambda_2) < 0$$

$$tc_1 = \operatorname{acos}\left(\frac{\sin(L_2) - \sin(L_1)\cos(\alpha)}{\cos(L_1)\sin(\alpha)}\right)$$

$$else$$

$$tc_1 = 2\pi - \operatorname{acos}\left(\frac{\sin(L_2) - \sin(L_1)\cos(\alpha)}{\cos(L_1)\sin(\alpha)}\right)$$

$$endif$$

Alternative calculation:

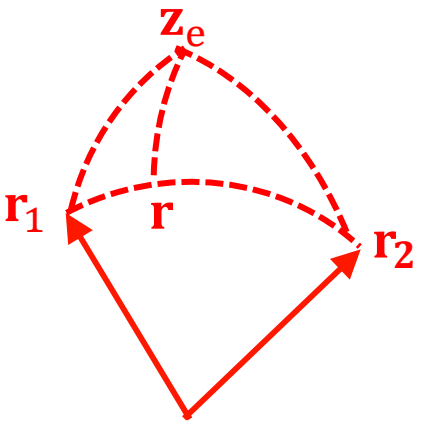
$$tc_1 = \operatorname{mod}\left\{\operatorname{atan}\left(\frac{\sin(\lambda_1 - \lambda_2)\cos(L_2)}{\cos(L_1)\sin(L_2) - \sin(L_1)\cos(L_2)\cos(\lambda_1 - \lambda_2)}\right), 2\pi\right\}$$

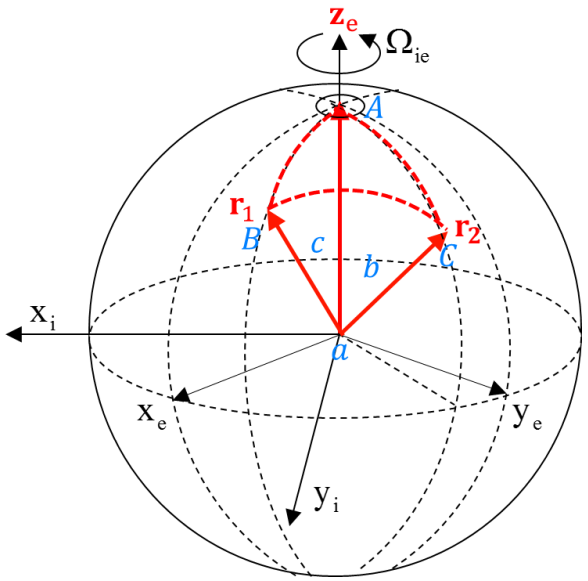
Implement using “atan2”

Great Circle Route Computation

- Given a longitude λ find latitude L:

$$L = \operatorname{atan}\left(\frac{\sin(L_1)\cos(L_2)\sin(\lambda-\lambda_2) - \sin(L_2)\cos(L_1)\sin(\lambda-\lambda_1)}{\cos(L_1)\cos(L_2)\cos(\lambda_1-\lambda_2)}\right)$$





Great Circle Route Computation

- Find latitude and longitude based on a fraction f of the distance between start- and end-point

1: $A = \frac{\sin((1-f)\alpha)}{\sin(\alpha)}$

2: $B = \frac{\sin(f\alpha)}{\sin(\alpha)}$

3: $x = A\cos(L_1)\cos(\lambda_1) + B\cos(L_2)\cos(\lambda_2)$

4: $y = A\cos(L_1)\sin(\lambda_1) + B\cos(L_2)\sin(\lambda_2)$

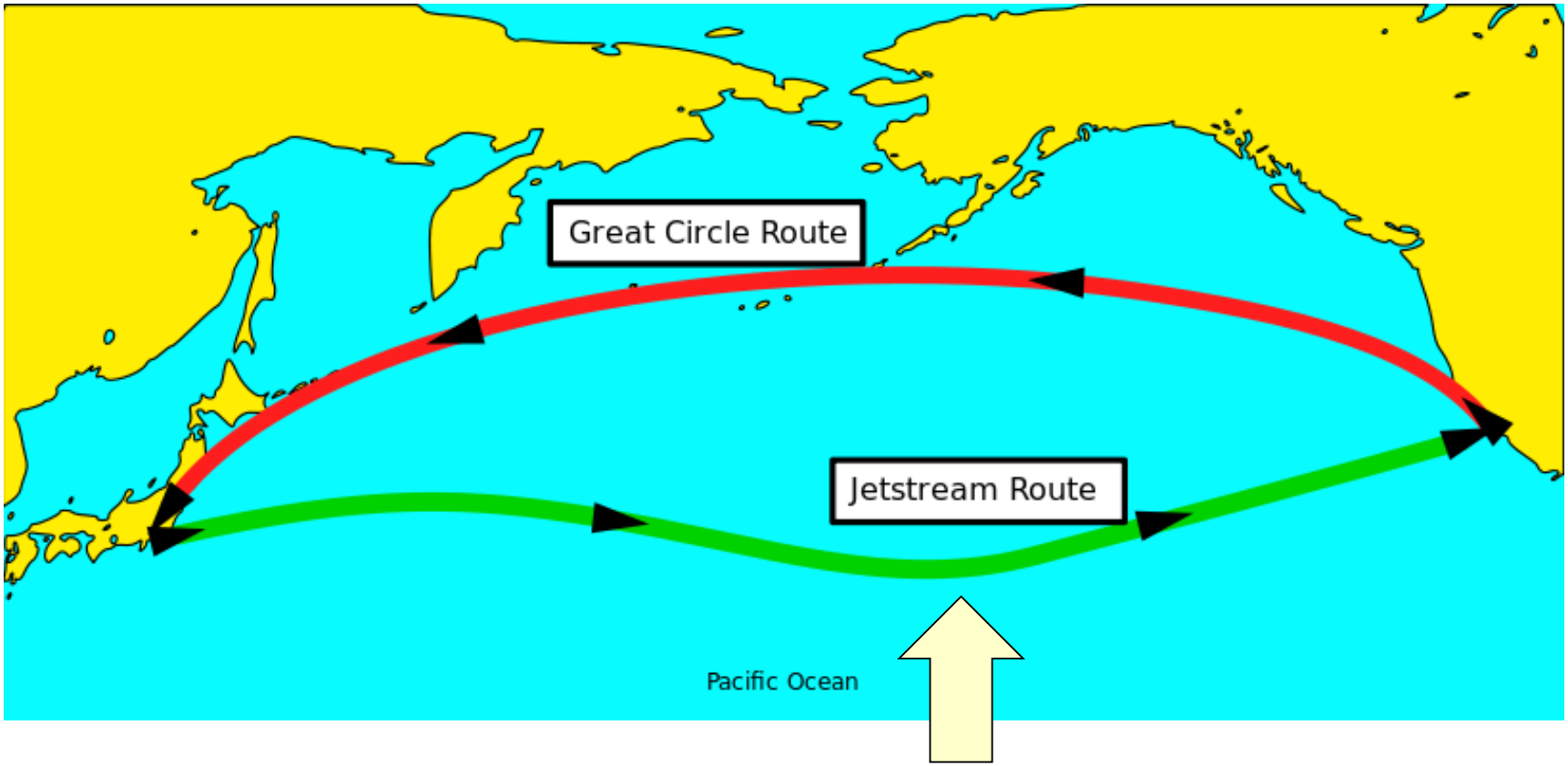
5: $z = A\sin(L_1) + B\sin(L_2)$

6: $L = atan\left(\frac{z}{\sqrt{x^2+y^2}}\right)$

7: $\lambda = atan\left(\frac{y}{x}\right)$

$$\mathbf{r}_i = R_e \begin{bmatrix} cos\lambda_i cosL_i \\ sin\lambda_i cosL_i \\ sinL_i \end{bmatrix} = R_e \mathbf{e}_i$$

Great Circle Route



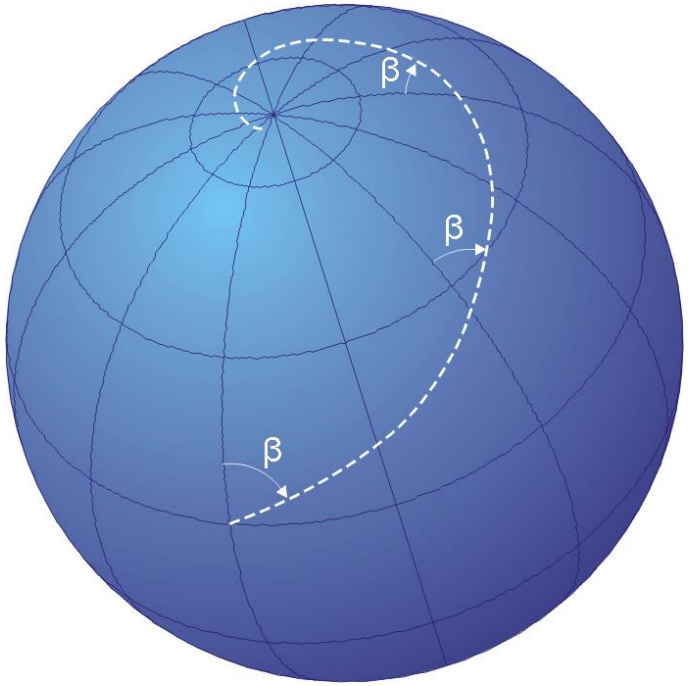
Other trajectory may be more optimal due to other factors just as wind, jetstream etc.

Now on an Ellipsoid …

- The curved space is now an ellipsoid rather than a sphere like the WGS 84 ellipsoid;
- Two example methods are described in:
  - T. Vincenty, "Direct and Inverse Solutions of Geodesics on the Ellipsoid wit Application of Nested Equations," Survey Review XXIII (176): 88–93, 1975.
  - C. F. F. Karney, "Algorithms for Geodesics," Journal of Geodesy, 87(1), 43-55, 2013.
- Both available on my website along with Matlab code for them.
- And, of course, the Mapping toolbox includes them all

Rhumb Line Navigation

- Great circle navigation:
  - Shorted distance
  - Bearing is not constant
- Rhumb line navigation:
  - Bearing is constant
  - Not shortest distance



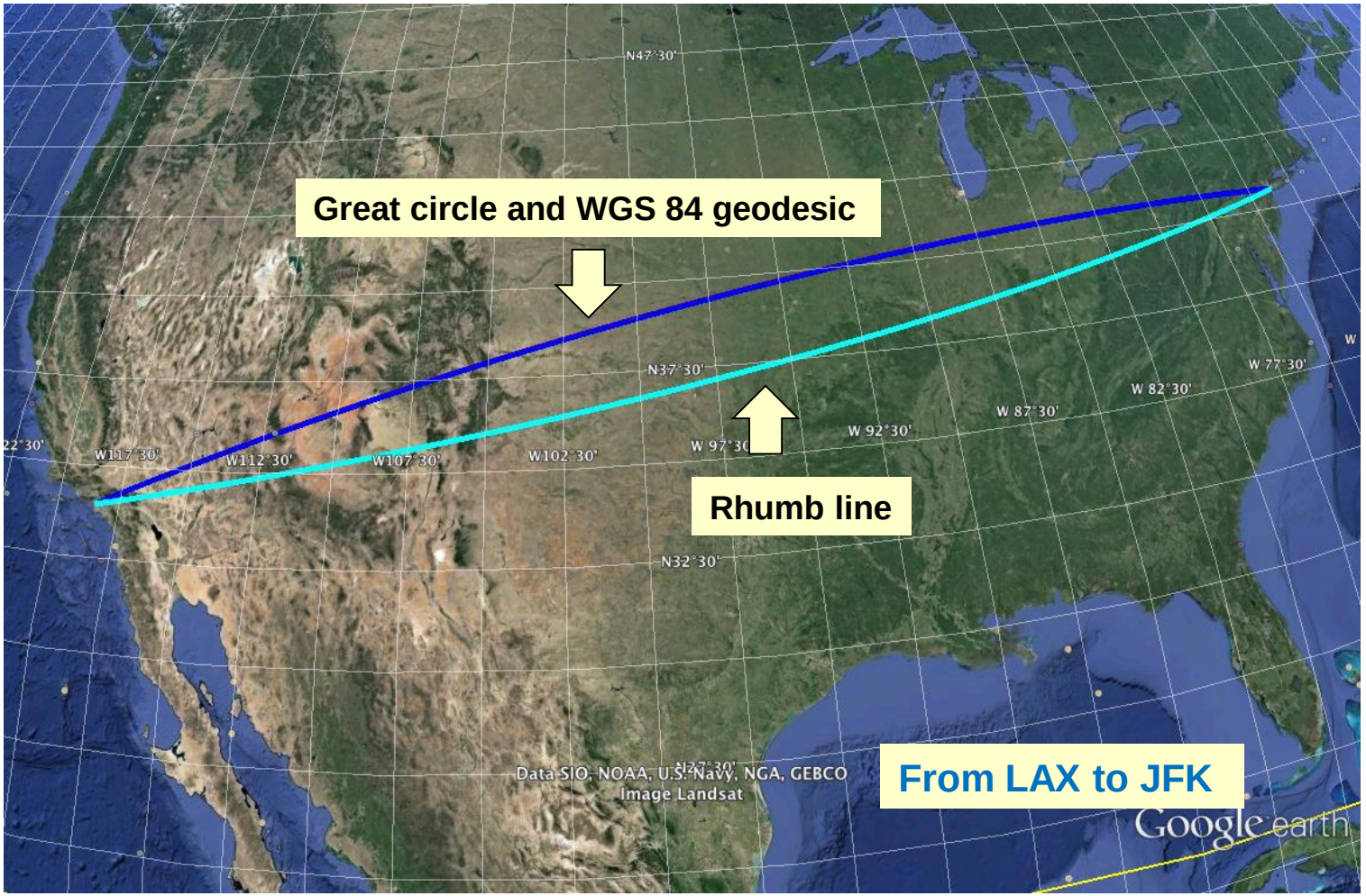
http://en.wikipedia.org/wiki/File:Loxodrome.png

Given two points (L_1, λ_1) and (L_2, λ_2) the true course between both is given by:

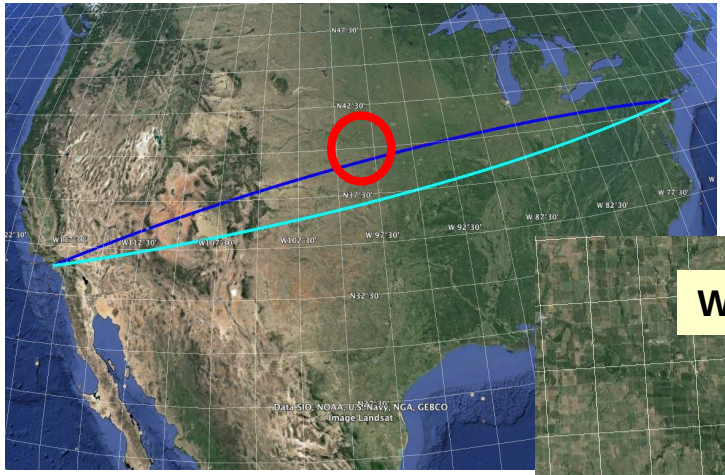
$$tc_1 = mod\left\{atan\left(\frac{\lambda_1 - \lambda_2}{log\left[\tan\left(\frac{L_2}{2}+\frac{pi}{4}\right)/\tan\left(\frac{L_1}{2}+\frac{pi}{4}\right)\right]}\right), 2\pi\right\}$$

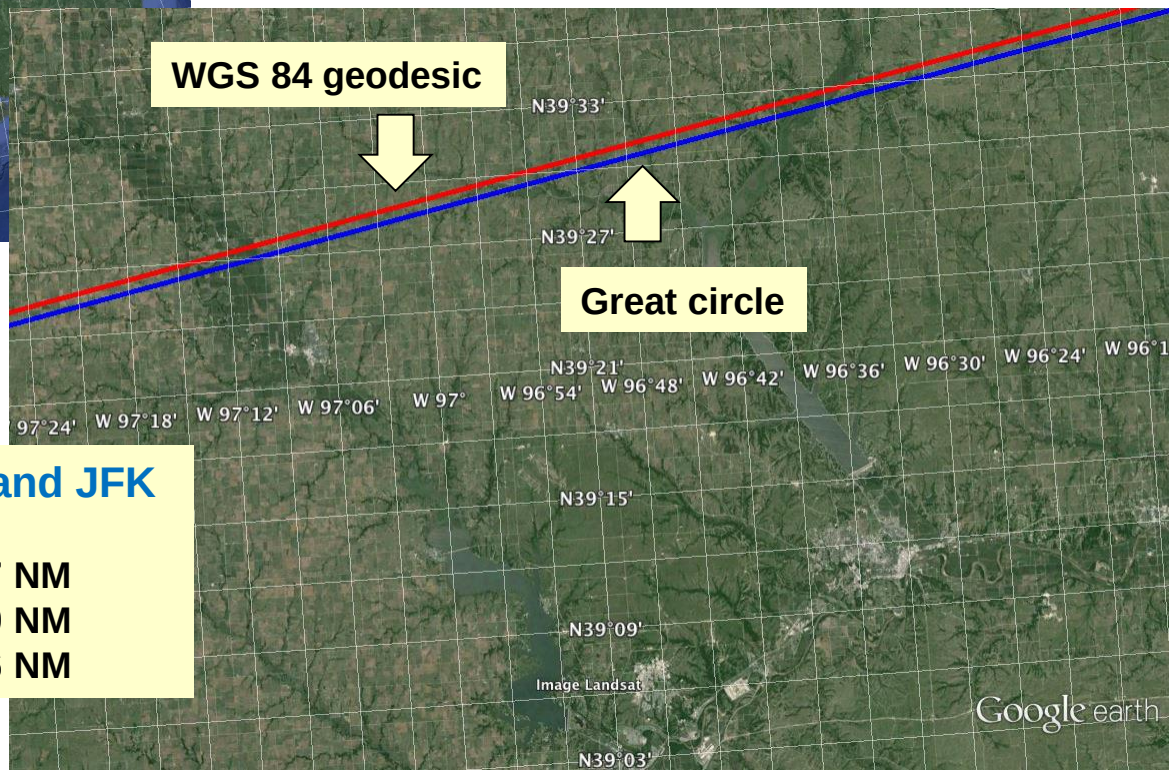
See also Matlab Mapping Toolbox

Great Circle, Ellipsoid, Rhumb Line



Great Circle, Ellipsoid, Rhumb Line





Distances between LAX and JFK

Great circle:	**2,143.7 NM**
WGS 84 geodesic:	**2,149.9 NM**
Rhumb line:	**2,164.6 NM**